APPLIED DESCRIPTIVE GEOMETRY

WITH

DRAFTING-ROOM PROBLEMS

BY

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PREFACE TO THE SECOND EDITION

The second edition of this text has been undertaken largely because of the many valuable suggestions that have been received from teachers who have been using the text since its first publication. These suggestions have been given the utmost attention by the author and have been adopted in almost every instance.

A small amount of mining material and an entire group of mining problems have been introduced. Approximately 15 per cent of the drafting-room problems in Chapter IX are new or have new data. A more logical transition has been made between Chapters II and III. Some statements in the text material have been clarified and illustrations have been added. A short appendix has been included, giving two tools which will be found to be very useful in problem solution.

The author wishes to express his sincere appreciation to his colleagues at the University of Washington, especially to Professor Tymstra, and to those professors at other schools whose personal interest and kindly cooperation have been such a wonderful help to him. It is his hope that this second edition will enable us all to obtain better results with less effort and more pleasure.

FRANK M. WARNER.

SEATTLE, WASH.,
May, 1938.

PREFACE TO THE FIRST EDITION

Perhaps no other subject in the entire engineering curriculum has been discussed, revised, and rewritten as many times as the subject of descriptive geometry. Yet this constant activity, on the part of the teachers of this subject, has produced no appreciable change in the array of fundamental principles that are presented. The very fact that these principles have withstood the test of time bears mute testimony to their practical value and to their completeness. Great credit is due to Gaspard Monge and to those authors who have followed him for their presentation and preservation of such useful and practical material.

The very usefulness and practicability of this subject have often been obscured by failure to relate its principles to actual engineering experience and the methods of the drafting room. Overcoming this fault brings into operation the first principle of education, *i.e.*, the arousing of student interest. If a student is taught how to find the angle a line makes with a plane, he sees little use for that information and therefore he will have little interest in it. But if he is given the problem of designing a casting to fit on an airplane wing for which he sees that he must be able to find the angle mentioned, he will speedily develop a keen interest in that same problem.

The foregoing simple illustration furnishes the reason why this book has been written. It is not intended to be a text on descriptive geometry, even though it necessarily reproduces most of the material ordinarily included in that subject. The chief objective of the author in offering this text to the profession is to place the major emphasis on the application of principles that have long been well known. These principles are wonderful tools for making certain graphical solutions. The best way to learn how to use a tool is to spend less time looking at the tool and more time using it.

The method followed is to start the student out with simple three-view orthographic projection. Then he is required to draw other objects, for which he will see that it is necessary to

know how to draw any kind of an auxiliary view. After he has become thoroughly familiar with the methods of drawing an object in any view desired, the ordinary point-line-plane problems are introduced and are solved by the same orthographic methods. No mention is made of any quadrants or angles of projection; the solutions are all obtained by the change-of-position or draftsman's method. A chapter is then presented giving the revolution method for solving many of the same problems. The following chapter on the graphical solution of noncoplanar forces and vectors, which the author believes to be unique in its new application of very simple principles to the solution of noncoplanar structural problems, will have the student keenly interested. Then the usual problems on curved lines and surfaces are introduced, cone and cylinder problems being emphasized particularly because they occur most frequently in engineering practice.

The last two chapters are devoted entirely to problems. Chapter VIII gives over 400 practice problems arranged in 61 groups. As each principle is introduced in the text, reference is made to one of these groups, for practice problems. They are given without data, for working on the blackboard or at home, and the author intends them to be checked for method only. However, it is easy for an instructor to assign data to them if he so desires.

Chapter IX is the main part of the book, since it furnishes the opportunity to apply the principles in a practical way. The problems are all carefully laid out ready to assign to a class. They have all been tested, and many of them will require the student to do some clear thinking. His solutions should be checked carefully by an instructor, for correctness and clearness of work. These problems furnish the basis for many new ones that may be taken from the experience of any wide-awake instructor. With some slight changes they will be found to be sufficient for several years' use.

The method of problem solution, as it is presented here, is not new. It has been in use at the University of Washington for over 20 years and has given much satisfaction. Students have shown the proper interest and are eager to master principles that they know they will need. Draftsmen of many years' experience have taken the course and have shown an enthusiastic interest in the method.

It is the sincere hope of the author that other teachers of this subject may find in this text a method which will produce improved results through greater student interest. He will be happy to give further advice or suggestions to any one desiring them, and he will be equally happy to receive helpful suggestions or constructive criticism from those who have used the book.

Acknowledgment is made here of the valuable assistance rendered by his fellow instructors of the General Engineering Department, without whose kindly help and cooperation this book would not have been written. Especial gratitude is expressed to C. W. Harris, Professor of Hydraulics, who wrote the first manual presenting this method in 1913, to Prof. E. R. Wilcox for constant encouragement, and S. R. Tymstra for proofreading and for many valuable suggestions for practical problems.

FRANK M. WARNER.

SEATTLE, WASH.,
February, 1934.

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CHAPTER IX

APPLIED DESCRIPTIVE GEOMETRY

CHAPTER I

ORTHOGRAPHIC DRAWING

1.1. Introduction.

Orthographic drawing is one of the most important subjects in the entire engineering curriculum. It bears the same relation to the engineering profession as the subject of English bears to our daily life; it is the means by which engineers communicate ideas. When it is properly used, this language of drawing is understood at once by engineers of all nations.

Drawings are not only convenient to the engineer; they are practically indispensable. A single drawing may sometimes contain a sufficient number of ideas to require an entire volume for their adequate expression in words.

Most drawings are made for the purpose of transmitting an idea to someone else. The draftsman who makes the drawing should, first of all, be able to visualize the object he wishes to draw, and his mental picture of this object should be perfectly clear. He must also have the ability to record this mental picture or idea on paper as a drawing, in such form that it will be perfectly clear to anyone skilled in the art of reading drawings.

Other drawings are made for the purpose of making certain graphical determinations in the solution of a space problem. These are more in the nature of layout drawings and are not always made for the purpose of transmittal to other people for reading. They may be made for the sole purpose of solving a calculation problem graphically.

A well-trained engineer is thoroughly familiar with both kinds of drawings. He knows how to make proper commercial drawings, and he knows how to solve any drafting-board problem quickly and correctly. He also appreciates the work involved in making a drawing and the limits of accuracy which may be

reasonably expected. Above all, he knows how to read drawings, because every job with which he will ever come in contact will be estimated, contracted for, built, inspected, bought, sold, operated, or maintained through the medium of drawings.

The principles of drawing and the drafting problems presented in this text are intended to give a logical training and a practical experience in orthographic drawing. A thorough understanding of these pages develops in the student confidence in his ability to visualize space problems, to make proper orthographic drawings, to construct a correct graphical solution of any space problem, and to read and to understand all types of drawings.

2.1. Descriptive Geometry.

Descriptive geometry is the subject which teaches how to make graphical solutions of space problems by the use of the same principles of orthographic drawing that are used in making simple views of an object. More briefly, descriptive geometry is orthographic drawing applied to the solution of more advanced space problems. The two should not be considered to be two different subjects which require the use of different tools or of different thinking processes. The same elementary principles which are used by an engineer to completely describe an object by orthographic views may be used to solve the more complicated problems in descriptive geometry. The reader should understand, at once, that descriptive geometry, as it is taught in this text, furnishes a method of solution for many practical problems, which requires the use of the identical principles he has already learned in his elementary drawing.

The material to be presented cannot be thoroughly understood without a proper knowledge of elementary drawing. There are many methods for teaching orthographic drawing, all of which have the same fundamental basis but differ in a few minor respects. In order that all readers of this text may become familiar with the conception of orthographic drawing upon which the whole book is based, the rest of this chapter is devoted to an explanation of orthographic views and the relationships which exist between them.

3.1. Change-of-position, or Direct, Method.

If it is desired to draw different views of an object, the draftsman first imagines it to be placed in some definite position. This

position is usually the one the object would naturally occupy. An engine base, a bridge, a truss, or a column footing would always be imagined to be resting in their natural position. Small castings, however, may be imagined in any desired position, in order to have the views of the drawing show the object in the easiest way and to the best advantage. After the object has once been placed in some definite position, the draftsman never imagines it to be moved or turned around. If he wishes to see a different side, he simply imagines himself walking around it in order to occupy a different position in space. The observer changes his position in order to look directly at the portion of the object which he wishes to see. Thus, this method is called the change-of-position, or the direct, method of drawing.

4.1. Definitions.

1. *Orthographic projection* means "right-angle projection" and is a method of drawing which uses parallel lines of sight at right angles to an image plane.
2. *A line of sight* is a straight line from the eye of the observer to a point on the object. Since all lines of sight for a given view are parallel, the eye of the observer is either at an infinite distance away or it occupies a different position when looking at each point on the object.
3. *The image, or picture, plane* is the plane on which an orthographic view is projected. It is always perpendicular to the lines of sight for any view, and it is always between the observer and the object.
4. *A top, or plan, view* is an orthographic view for which the lines of sight are vertical and for which the image plane is level.
5. *An elevation view* is an orthographic view for which the lines of sight are horizontal and the image plane vertical.
6. A *folding line* is the line of intersection between two image planes and is the line one image plane is folded on to bring it into the plane of the other image plane.
7. *Projection lines* are straight lines at right angles to the folding line which connect the projection of a point in one view with the projection of the same point in another view. They are very necessary for obtaining views, but they are not always shown on a drawing.

Figure 101 is a pictorial drawing illustrating all of these seven definitions so as to give a correct conception of their relations to each other in space. The object itself is shown shaded and

behind all three image planes. Attention is called to the fact that each view of the object is right on the image plane for that view. In other words, the image plane is the plane of the paper itself on which the actual drawing is made. A most careful study of the foregoing definitions and illustration should be made.

5.1. Folding the Image Planes.

All drawings in practice, including all views, are actually made in one plane, which is the plane of the paper or drawing board.

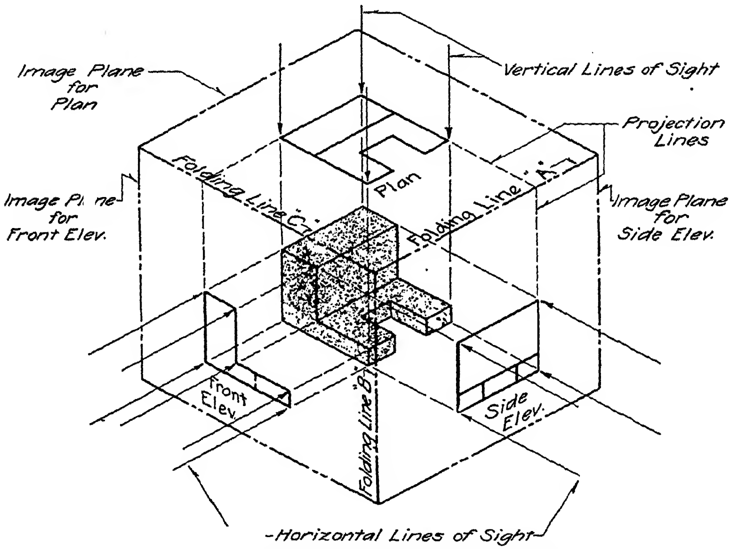


FIG. 101.—Pictorial view illustrating the definitions.

Hence the image planes, which are shown in Fig. 101, must all be brought into the same plane in order to occupy the position where the views are actually drawn by the draftsman. This may be done in two different ways.

The first way to fold the image planes is to imagine the plan image plane to remain stationary and the other two image planes to be folded about the folding lines A and C until they lie in the same plane as the plan image plane. This will bring the three views into the positions shown in Fig. 102, with the side elevation projecting from the plan view.

The second way to fold the image planes is to imagine the front image plane to remain stationary and the other two to be

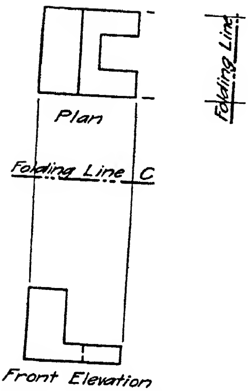


FIG. 102.—The views folded into the plane of the plan.

into the position shown in Fig. 103, with the side elevation projecting from the front elevation.

In either of these methods the resulting views themselves are identical, but the side elevation simply occupies a different posi-

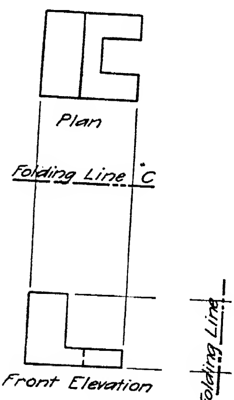


FIG. 103.—The views folded into the plane of the front elevation.

tion on the paper. Both methods are absolutely correct and both are largely used by draftsmen on various kinds of commercial drawings. The author favors the first method for

beginners, for the simple reason that the plan view seems to be the *basic view.* The plan view has vertical lines of sight; these can have only one possible direction, and therefore there can be only one possible plan view. It is assumed that vertical lines of sight always look down and not up, in accordance with standard commercial practice. Lines of sight for elevation views are level and may have an infinite number of directions. Therefore we may have an infinite number of elevation views of any object. Since, then, there can be only one plan view, but any number of elevation views, it seems logical to treat the plan as the basic view and to take all elevation views from the plan. Experience has shown, too, that it is easier for most students, at first, to take elevation views from the plan view. After more experience has been gained, the view that will solve a problem in the simplest and clearest way, or which will better suit the available space on the paper, should be used.

6.1. Placing the Views.

Every piece of structural work, small casting, or machinery part, which an engineer intends to build must first be drawn by a draftsman. This drawing must be an accurate and complete description of the article to be built, both as to its shape and as to its size. In order that this drawing may be interpreted in exactly the same way by everyone who reads it, there must be a universally recognized system for placing views. The system which is used by practically every drafting room in the United States is the one which is followed in this text.

In this system the image plane is always considered to be between the observer and the object, as has been shown in Fig. 101. If this rule is adhered to and if the image planes are folded as explained in Section 5.1, the drawing automatically becomes a third-angle drawing, which conforms to the best commercial drafting-room practice in our country. Without explanation of the four-quadrant conception of drawing, which is fast becoming obsolete, a third-angle drawing may be defined as one in which a view is always placed on the same side of an object as the observer of the object. The explanation which has just been given is illustrated in Fig. 104.

An observer, who imagines himself standing south of the house to view it, places this view on the south side of the plan, that is on the same side from which the object was viewed.

This is the most natural placement for any view, and *this method should be strictly adhered to* in all drawing problems.

As an aid in understanding this arrangement more clearly, it is most helpful to copy the drawing of Fig. 104 on a sheet of paper and to cut out the two corners on the dashed lines as indicated. Then fold the image planes for the three elevation views down into their proper positions in space, until they are at right angles to the plan image plane. Then stand these image planes on a table and walk around them, keeping your lines of sight level. Imagine the house itself to be in behind the four image planes. Notice whether or not the elevation views seem to be correctly placed.

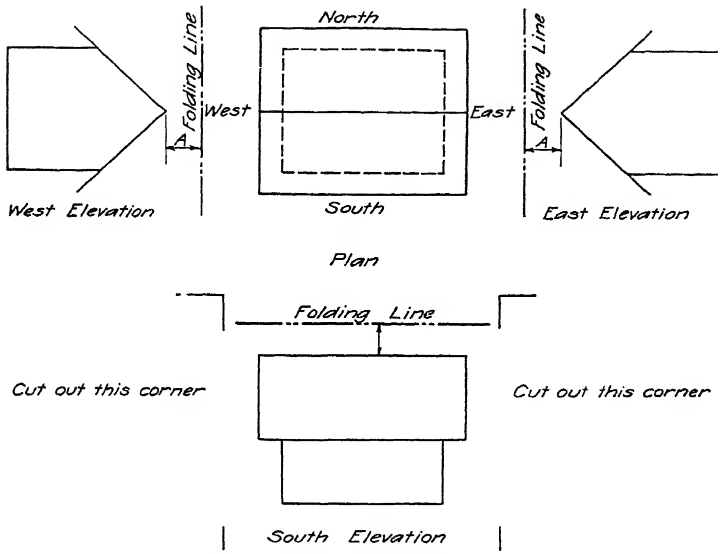


FIG. 104.—Accepted method of placing views.

7.1. Distance from the Folding Line.

In Fig. 104, special attention is called to three self-evident facts which are most important:

1. The highest point on the house, or the ridge, is always toward the plan in all elevation views.
2. The house is the same height in all elevation views.

3. In all elevation views the ridge is the same distance below the folding line. This was to be expected, since the ridge itself, in space, is the distance A below the plan image plane and hence will appear the distance A below the folding line (which is the plan image plane appearing as an edge) in any view having level lines of sight. This being true for these three elevation views, it is likewise true for all elevation views. It is also true for any other point on the house, as well as for the ridge.

These facts may be stated more clearly in the form of a rule.

Rule 1. Any point on an object will appear the same distance below the folding line in all elevation views.

8.1. Notation.

The solution of more difficult problems in drawing often requires the use of more than two or three views. A very simple

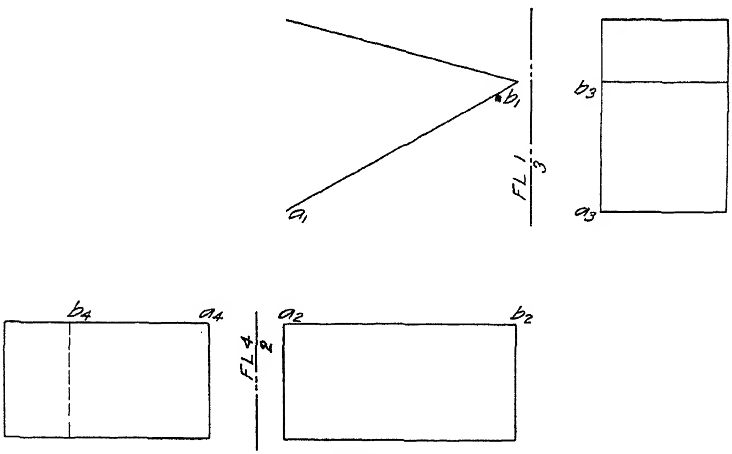


FIG. 105.—Notation system.

system of numbering the views and folding lines is a great aid to the draftsman in keeping the drawing clear, and it eliminates the necessity of labeling the views. Since the plan is considered to be the basic view it is always given number 1. The next view to be drawn is given number 2, and all other views are then numbered in the order in which they are drawn, as shown in Fig. 105. The numbers of the views are placed on each side of the folding line as shown. The folding line, therefore, carries

the numbers of the views it lies between. In some problems it is also convenient to letter each point in each view. Lowercase letters are always used and they are given a subscript number which is the same as the view number. Anyone who is familiar with this notation can tell at a glance which is the plan view, and what the order of solving the problem was, by the sequence of the view numbers.

Capital letters are used in referring to a line itself in space, as the corner AB of the wedge in Fig. 105.

The folding lines are shown with dashes, one long and two short. They are shown as rather heavy lines in this text, in order to distinguish the views better. However, for accurate pencil solutions, the folding line should be drawn as a fine line but with the same notation.

The notation to be used for the object lines and solution lines is not specified, because an exact specification becomes too complicated and reduces the opportunity for the student to exercise his own judgment. In general, it may be said that construction lines should always be very fine and accurate; the object lines should be more pronounced. Judgment must be used to make the drawing clear and understandable at a glance.

9.1. Practice Problems.

See Chapter VIII, Group 1.

10.1. Auxiliary Elevation Views.

The statement has already been made that elevation views have level lines of sight and that level lines can have an infinite number of directions, making possible an infinite number of elevation views. The common elevation views are the front, the rear, the right-side, and the left-side views. Any other view using level lines of sight is called an auxiliary elevation view. Often it is necessary to draw such a view in order to see a definite part of an object in its true size and shape. By use of the principle of Rule 1 and the fact that the lines of sight for any view are parallel, any possible auxiliary elevation view is as easily drawn as a side elevation view.

In Fig. 106, it is desired to obtain a view of the bracket with lines of sight perpendicular to the face A as shown by the arrow. Then all the lines of sight for this view are parallel to this arrow. The point C on the object, in this view, will appear on a definitely

fixed line of sight, or projecting line, and it must be A distance below the folding line, giving c_3. All other points on the object are obtained in this view in the same manner.

Any other auxiliary elevation can be drawn as easily by locating each point on its proper projecting line at the correct

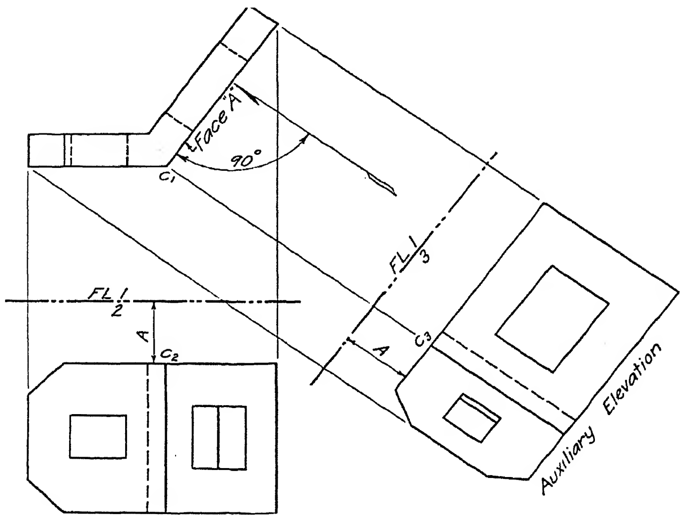


FIG. 106.—Auxiliary elevation.

distance below the folding line. If any difficulty is experienced in visualizing any part of this explanation, a paper drawing should be made and cut out and folded as suggested for Fig. 104.

11.1. Practice Problems.

See Chapter VIII, Group 2.

12.1. Inclined Views.

An inclined view is any view for which the lines of sight are neither vertical nor horizontal. In other words, the lines of sight for any inclined view must be sloping or inclined.

The object in Fig. 106 might have been placed in a different position for purposes of drawing, as in Fig. 107. Attention is called to the fact that in Fig. 107 the view marked 2 is a front elevation view and *not* the plan, although it is identical with the plan of Fig. 106. The position an object occupies in space must always be kept clearly in mind. The face A is now an

inclined plane and, in order to look with lines of sight perpendicular to it, inclined lines of sight will have to be used which are parallel to the arrow. The point *M* on the object lies in a fixed line of sight. But, in this case, the distance *B* from the folding line is obtained from the plan view because the point *M* is actually *B* distance behind the front image plane. In both the inclined view and the plan the folding line is the front image

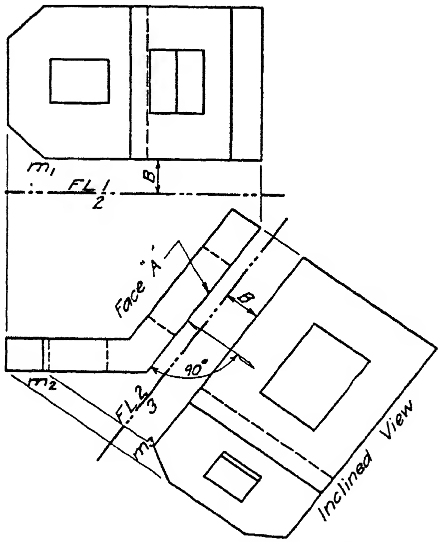


FIG. 107.—Auxiliary inclined view.

plane appearing as an edge. This determines m_3 in the inclined view. All other points on the object are located in the inclined view in a similar manner. By this method an inclined view is drawn as easily as any elevation view. Once more the drawing should be made and the paper cut and folded, that there may be a clear understanding that the *B* distance is the correct one to use.

13.1. Practice Problems.

See Chapter VIII, Group 3.

14.1. Additional Inclined Views.

In Fig. 107 the inclined view was taken directly from the front elevation. Sometimes it becomes necessary to draw an inclined

view from an auxiliary elevation, as in Fig. 108. The method is just the same as that shown in Fig. 107, but in this case the distance C from the folding line in the inclined view is taken from the plan view as shown on the drawing. It is an entirely different measurement from the B distance of Fig. 107. In this case it is taken from the plan as shown because the center

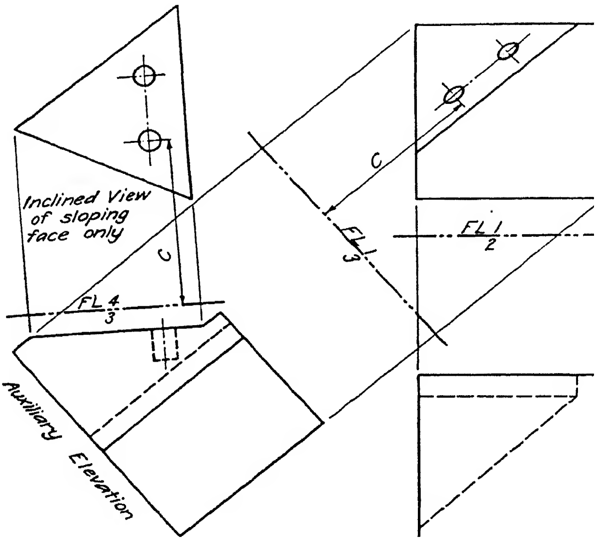


FIG. 108.—Inclined view from an auxiliary elevation.

of the nearest hole on the object is actually C distance behind the image plane for the auxiliary elevation.

The drawing should again be made, and the paper cut and folded, in order to see clearly why this C distance is the correct one to use. However, if the drawing is even turned, temporarily, so that the auxiliary elevation occupies the position of the front elevation, it will be very evident why the C measurement is correct.

Inclined views may also be taken from any other inclined views by following the same procedure outlined for the two preceding problems.

15.1. Practice Problems.

See Chapter VIII, Group 4.

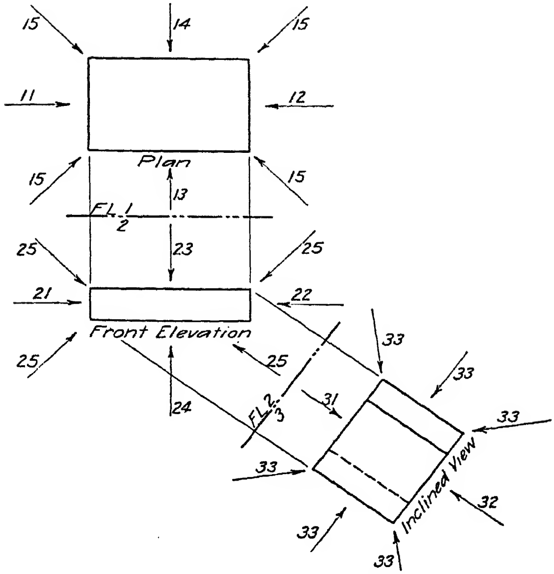


FIG. 109.—Lines of sight for all possible orthographic views.

TABLE I

View taken from	Arrow numbers	View obtained
Plan (1)	11 12 13 14 15	Left-side elevation Right-side elevation Front elevation Rear elevation Auxiliary elevations
Front elevation (2)	21 (Same as 11) 22 (Same as 12) 23 24 (Seldom used) 25	Left-side elevation Right-side elevation Plan Bottom view Inclined views
Inclined view (3)	31 (Same as 13) 32 (Same as 14) 33	Front elevation Rear elevation Inclined views

16.1. Summary of All Possible Orthographic Views.

1. **Plan View.**—If all vertical lines of sight are assumed to be looking down, there is only one possible plan view of an object fixed in space. This is the *basic view*.

2. **Elevation views** are derived from the plan view, from other elevation views, or from inclined views. From a given plan view we may take an infinite number of elevation views. From a given elevation view we may take only two other elevation views. From a given inclined view we may take only two elevation views.
3. **Inclined views** are taken from elevation views or from other inclined views, but *never* from a plan view. From a given elevation view we may take an infinite number of inclined views. From a given inclined view we may take an infinite number of inclined views.

Figure 109 illustrates all the different possible orthographic views which have just been explained. The arrows indicate the different possible directions for lines of sight, and each arrow is numbered and referred to in Table I. This table shows clearly what kind of view would be obtained by using lines of sight parallel to each numbered arrow shown in Fig. 109. After carefully studying Table I and Fig. 109, one should always be able to name correctly any view which is taken from any other view.

17.1. Related Views.

Views are said to be *related* to each other when:
1. Their image planes make 90 degrees with each other in space.
2. They have folding lines between them.
3. The two views of any point lie on the same projecting line which is at right angles to the folding line between the views.

All three of the foregoing conditions must be satisfied if two views are related. Views which are related to a common view are not related to each other, but they do have a common dimension which is at right angles to the folding line in both views. In Fig. 110, views 1 and 4 are related to view 2. But view 4 is not related to view 1, because corresponding points on the object will not project in parallel lines between these views and because there is no folding line between these two views.

The drawing of Fig. 110 gives a complete summary of the relationships between the various possible views. The table in Fig. 110 shows just which views are related to each other. For instance, the only views that are related to the plan view 1 are views 2, 3, and 5, all of which must be elevation views.

The correct measurements to take in order to obtain any new view are also shown in Fig. 110. They may be summarized in another rule.

Rule 2. In all views that are related to a common view, the object, or any point on the object, is the same distance away from the folding line.

If the finger is placed on any orthographic view, all views which project from it (or are related to it) are the same distance from the

Views	Are Related to	
No.	No.	Name
2-3-5	1	Plan
1-4	2	Front Elev.
1	3	Side Elev.
2	4	Inclined
1-6	5	Auxiliary Elev.
5	6	Inclined

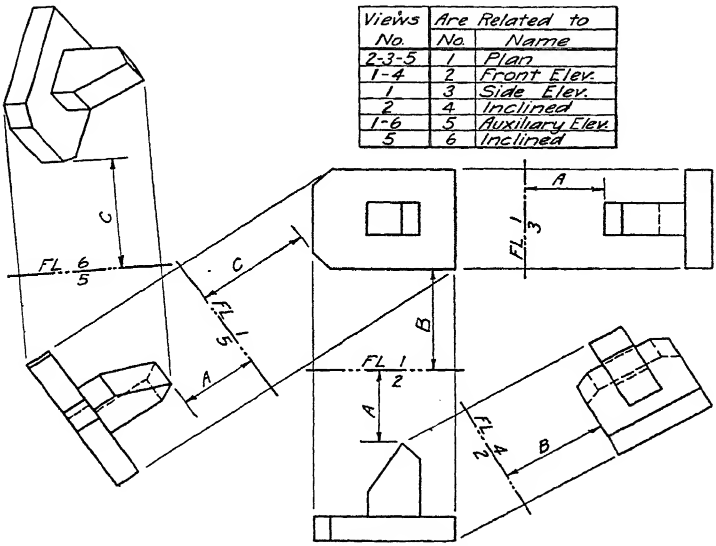


Fig. 110.—Related views.

folding line. This fact may easily be checked by placing a finger on some view, such as view 5. Views 1 and 6 are both related to view 5 and they both lie *C* distance away from the folding line.

Another method of checking the measurement from the folding line is temporarily to consider any view as a plan. Then all views related to that view are, temporarily, elevation views, and the measurement from the folding line seems more easily checked. However, this last method is not recommended except in cases of extreme difficulty in visualization.

18.1. Purpose of Folding Lines.

The use of image planes and folding lines which has just been explained is the most fundamental method for clearly

visualizing the relationship between views and is the proper way to measure the size of an object in a new view which is to be drawn. In shop drawings and detail drawings of simple objects which require only two or three views for describing them completely, the folding line is not usually shown, although it may be imagined to be there. For such drawings it is perfectly proper and it is good commercial practice to omit the folding line entirely.

However, in the more complicated problems of descriptive geometry which follow in this text, it is imperative that there be some place from which to make measurements and some standard way of measuring. The folding-line system provides this means of measuring and is therefore followed throughout this text. It should be used by the student in all the problem solutions, for it will prove to be very convenient and will be a very great help in making the solutions.

CHAPTER II

FUNDAMENTAL AUXILIARY VIEWS

1.2. Necessity for Auxiliary Views.

The preceding chapter has shown that it is possible for an observer to stand at any desired place in order to view an object at any desired angle. It has also shown how the draftsman draws the different views on paper and keeps them properly related. The reason why it sometimes becomes necessary to view objects or problems from different angles will now be explained.

All commercial drawings must be dimensioned. Almost every object which is drawn is bounded by lines and planes. No line or plane can be dimensioned in any view unless that line or plane appears in its true size in that view. No work line on a steel drawing can be dimensioned in any view except in the view which shows its true length. No angular cut on a steel plate can be dimensioned unless shown in its true size. Therefore the draftsman must know how to obtain quickly that view of an object which will show it exactly the way he must see it in order to dimension it properly.

2.2. Four Fundamental Views.

The experience of the author and of many other engineers has proved to their entire satisfaction that the necessary views, as explained in Section 1.2, may always be obtained by drawing certain basic or fundamental views. Practically any object composed of lines and planes that an engineering draftsman would ever have to draw, can be drawn and dimensioned by the use of one or more of the four fundamental views listed below.

View A. Showing the true length of a straight line.
View B. Showing a straight line as a point.
View C. Showing a plane as a line or an edge.
View D. Showing a plane in its true size and shape.

In order fully to explain these four views, the fundamental conceptions of lines and planes must be considered first.

3.2. Lines. Definitions.

1. A *line* is the path of a moving point.

2. A *straight line* is the path of a point moving constantly in the same direction. Hereafter in this text it may be assumed that the word "line" implies a straight line unless otherwise designated. Since any two points determine a straight line, a line is usually designated by its two extremities if it

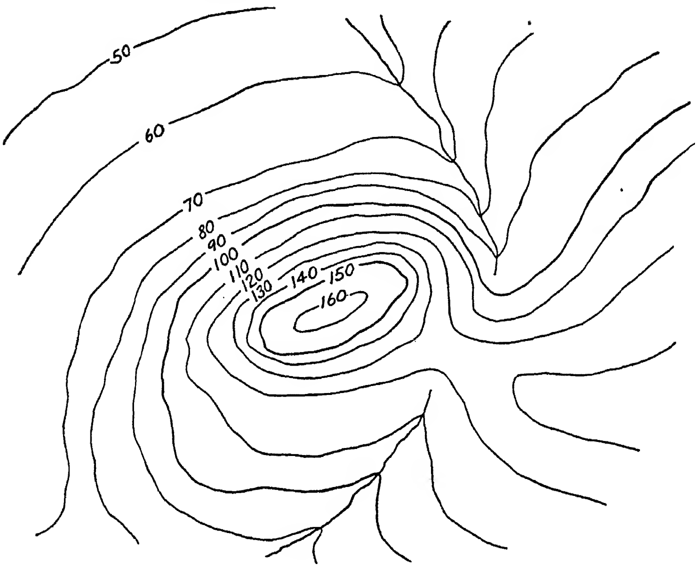


Fig. 201.—Contour lines on a map.

has a fixed length. Otherwise any two points on the line may be chosen at random for the purpose of locating the entire line in any other view.

3. A *level line*, or a *horizontal line*, is a line on which every point lies at the same elevation.
4. A *frontal line* is a line which lies parallel to the image plane for the front elevation. It may be level, vertical, or inclined and it always shows in its true length in the front elevation.
5. A *vertical line* is one that is perpendicular to a level plane. A line that is perpendicular to an inclined plane is not vertical. *Vertical* and *perpendicular* have quite different meanings.
6. A *contour line* on a map is a line, straight or curved, connecting a series of points which are at the same elevation or level. It is therefore a level line. Figure 201 shows a map with 10-ft. contour lines, which means that the contour lines are

shown at every difference of 10 ft. in elevation. A person who is at the 80-ft. level and who wishes to walk around remaining at the same elevation would have to follow the 80-ft. contour line.

4.2. Bearing.

The bearing of a line is the angle between its plan view and the plan view of a line running due north and south. The angle less than 90° is usually given for the bearing, and it may be given

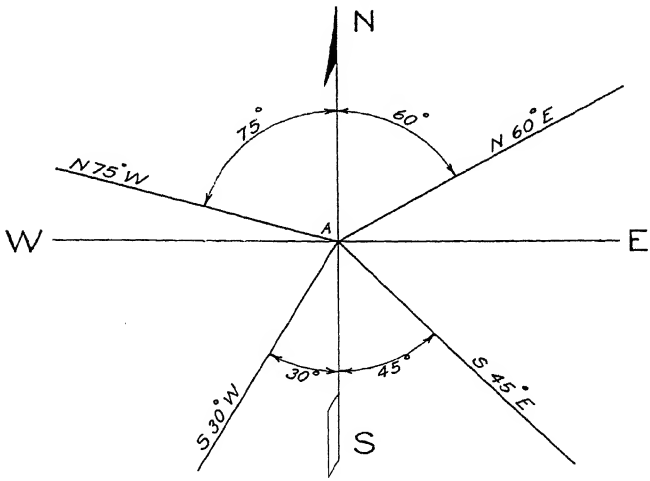


FIG. 202.—Bearing of a line.

from either the north point or the south. If the bearing of a line, from a given starting point, is said to be N 60° E, it means that the plan view of the line is turned away from the north line 60° toward the east, as is shown in Fig. 202. The bearing of a line is not at all affected by its slope. If the bearing of a line is given as N 75° W, the direction of the plan view is absolutely fixed regardless of whether the line is level or whether it has a steep slope. The bearing of any level line on the plane of a vein of ore is called the "strike." See Section 38.3.

5.2. True Length of a Line. View A.

A line must lie at right angles to the lines of sight for any view in order to appear in its true length in that view. In other words,

a line must lie parallel to the image plane for the view which shows its true length. Only level lines appear in their true length in the plan view, because the plan image plane is level and all lines that are parallel to it must also be level. In Fig. 203 the front elevation shows that the timber is in a level position and parallel to the plan image plane. Therefore the plan view shows the true length of this timber.

Any vertical line appears in its true length in all elevation views, because the image planes for all elevation views are vertical and hence any vertical line is parallel to them all. In Fig. 204

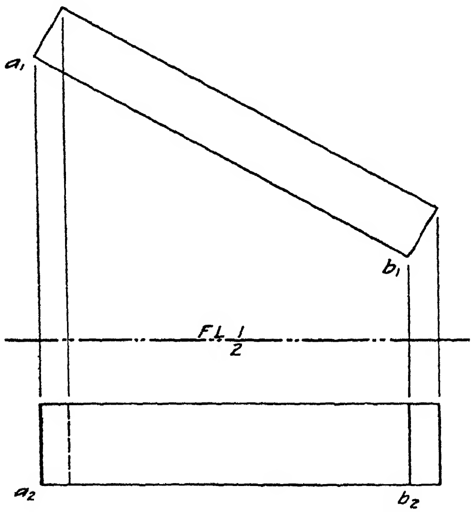


Fig. 203.—A level beam in its true length.

the vertical mast shows in its true length in all three elevation views because it is parallel to the image planes for all three.

In Fig. 205 are shown a plan and a front elevation of a portion of a hip roof. The hip rafter AB does not show in its true length in the plan because, by inspection, it is not level. It also does not show in its true length in the front elevation, because the lines of sight for that view are not at right angles to it. If its true length is desired, a new elevation view must be drawn with lines of sight at right angles to the rafter. This auxiliary elevation is easily determined by the use of parallel lines of sight and of the principle of Rule 1, Section 7.1. Also an inclined view may be

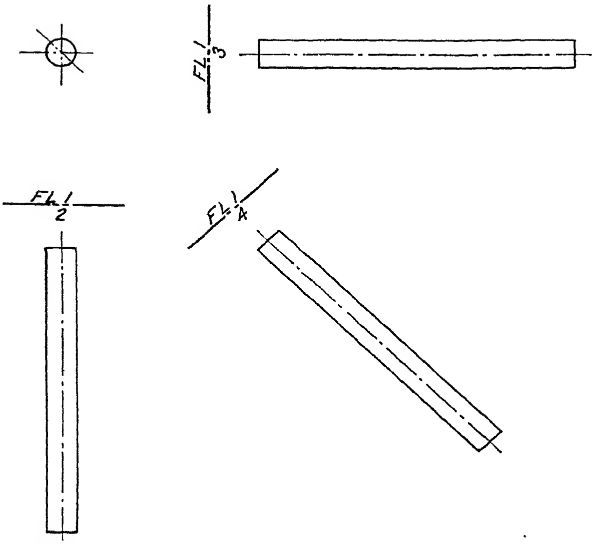


FIG. 204.—A vertical mast in its true length in all elevation views.

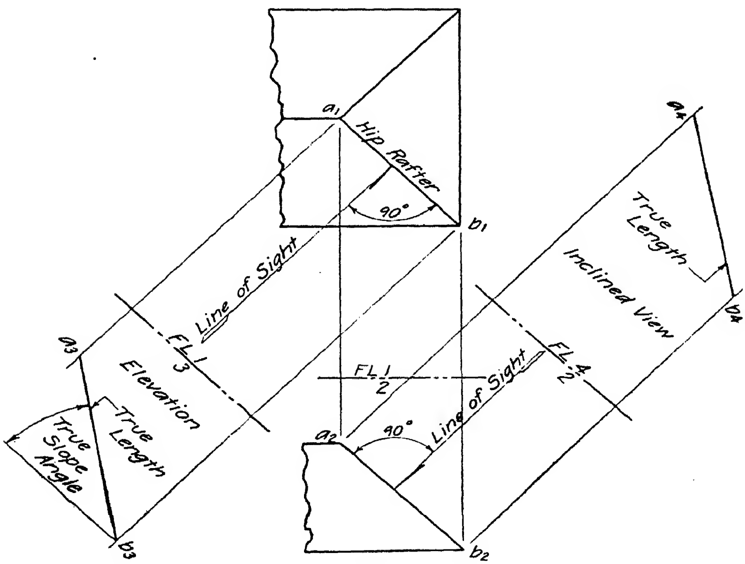


FIG. 205.—True length and true slope of a hip rafter.

drawn from the front elevation, with lines of sight at right angles to the rafter as shown by the arrow. This inclined view also shows the rafter in its true length, which should check exactly with the true length determined in the auxiliary elevation.

6.2. Practice Problems.

See Chapter VIII, Group 5.

7.2. True Slope of a Line.

The true slope of a line is the angle the line makes with a horizontal plane, as is shown in Fig. 206. The per cent grade of a road, considered as a centerline, is 100 times the tangent of the angle between the road and a horizontal plane, as is shown in Fig. 207. A road rising a vertical distance of 10 ft. in going a hori-

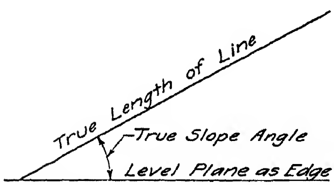


FIG. 206.—True slope angle.

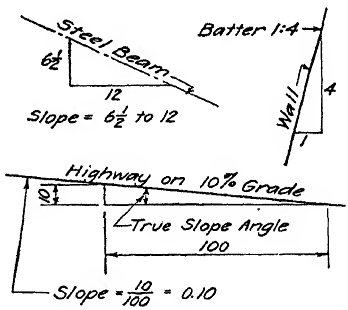


FIG. 207.—Methods of indicating slope.

zontal distance of 100 ft. is said to have a 10 per cent grade, or the tangent of the slope angle is 10/100. The most common engineering practice is to designate the amount of slope by the per cent grade, although it may be measured in degrees. The slope, or the bevel, of a steel beam is given as shown in Fig. 207, the longest side always being taken as 12. The slope of concrete walls which are almost vertical is sometimes called "batter" and may be indicated as shown or by just a lettered note in the plan view. All these methods for dealing with slope will be used in the problems of Chapter IX.

Since the slope angle between any line and a level plane lies in a vertical plane which contains the line, its true size can be seen only when viewed at right angles to that vertical plane, with

level lines of sight. Elevation views are the only views for which level lines of sight are used. Therefore the following rule may be stated

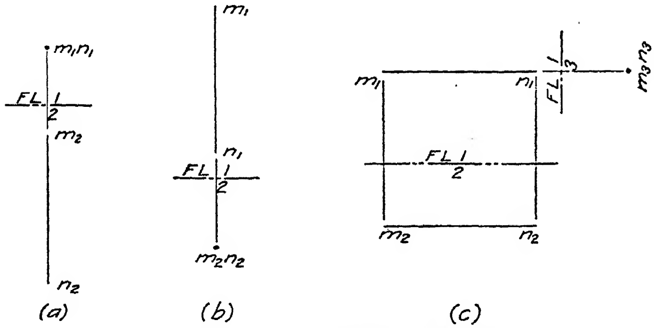


FIG. 208.—Views showing a line as a point.

Rule 3. The true angle of slope of any line can be seen only in the elevation view which shows the true length of the line.

In Fig. 205, again, the true slope of the hip rafter is seen in the auxiliary elevation view which shows its true length, and it is so marked in this figure.

Caution. An inclined view may show the true length but NEVER *the true slope of a line,* because it cannot show a level plane as an edge.

8.2. Practice Problems.

See Chapter VIII, Group 6.

9.2. A Line as a Point. View B.

If the lines of sight for any view are parallel to a line in space, that line appears as a point in that view. Figure 208 shows the line MN occupying three different positions in space. In Fig. 208 (a) the line shows as a point in the plan view because it is in a vertical position. In Fig. 208 (b) it shows as a point in the front elevation because it is a level line and lies parallel to the lines of sight for the front view. In Fig. 208 (c) it appears as a point in the side elevation because it is level and lies parallel to the lines of sight for the side elevation.

Figure 209 shows four views of a level line AB. The new elevation view 4 is drawn from the front elevation, and the line AB does not appear as a point in this view. But a new auxiliary

elevation, view 3, taken from the plan view does show the line as a point, because the line AB is in its true length in the plan. For view 3 the lines of sight are actually parallel to the line AB itself in space. For view 4 this is not the case.

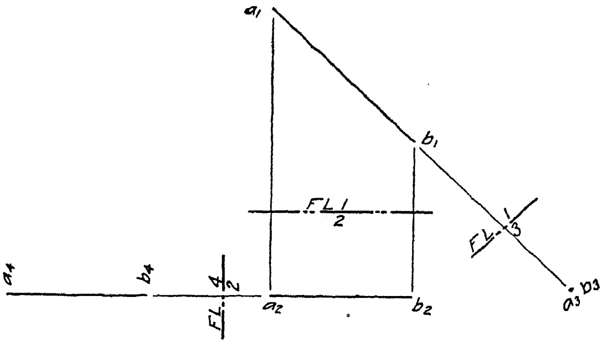


FIG. 209.—Showing that a line does not always appear as a point.

Now let the same line AB have the end A raised a little as in Fig. 210. Once more, view 4 is drawn from the front elevation and view 3 is drawn from the plan. But the line AB does not show as a point in either of these new views, if the principles of projection have been used correctly. The lines of sight for these new views only appear to be parallel to the line AB in space.

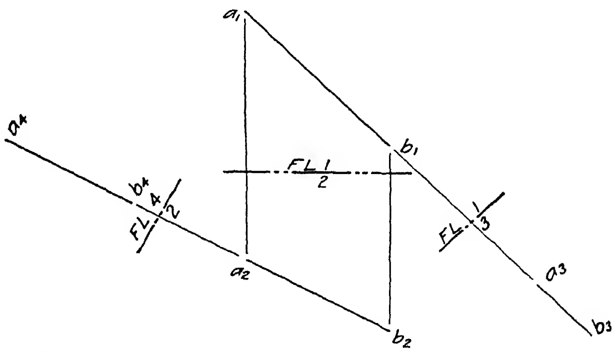


FIG. 210.—Showing when a line does not appear as a point.

Actually they are not parallel to the line AB, because that line is not in its true length in either view. With the line AB in the position shown in Fig. 210 it is impossible to draw any view from the plan or the front elevation which would show it as a point.

Before the line AB can be seen as a point in any view, a new view must be drawn showing it in its true length, as in Fig. 211, view 3. An inclined view 4 taken from the true length view 3 is found to show the line as a point. This may easily be proved by actually making the drawing and locating each point (A and B) in the new views by measurement as was explained in Chapter

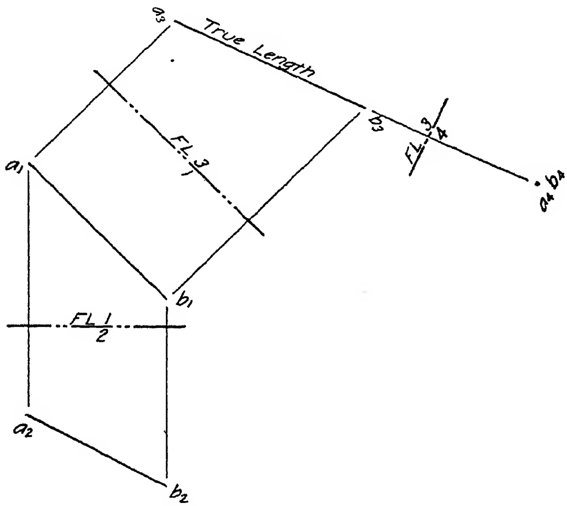


FIG. 211.—The condition necessary in order to see a line as a point.

I. It is self-evident that, since the line AB is inclined, we should have to use inclined lines of sight to see it as a point. This explanation is now summarized in a very important rule.

Rule 4. A line must appear in its true length in some view before a view may be drawn showing it as a point.

This rule should be checked by referring again to Figs. 208, 209, and 211, and observing that in every case the view that shows the line as a point is related to the view that represents the line in its true length.

10.2. Practice Problems.

See Chapter VIII, Group 7.

11.2. Planes.

A plane is a surface such that, if any two of its points are con-

nected by a straight line, that straight line always lies wholly on the surface; or every point in that line is on the surface.

A plane may be determined in space in three different ways, all of which are common in engineering work and are used in the problems in this text. These three ways are as follows:

1. By any three points not in a straight line, as in Fig. 212 (*a*).

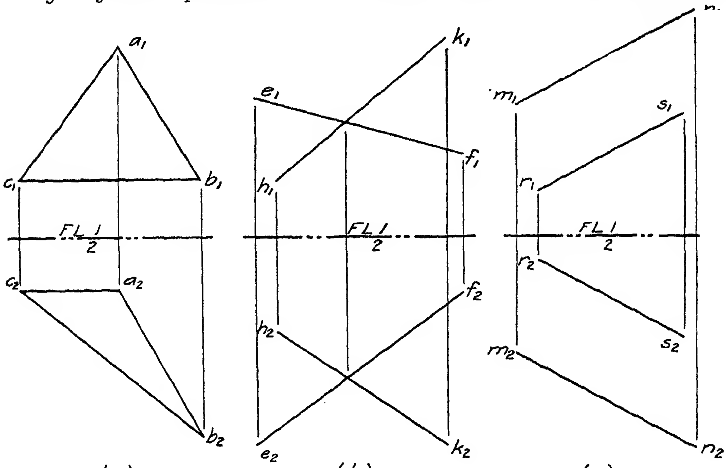


FIG. 212.—Three ways of representing planes.

2. By any two intersecting lines, as in Fig. 212 (*b*). It should be noted that two intersecting lines have one point in common that projects between views.
3. By any two parallel lines, as in Fig. 212 (*c*).

A plane which forms a limiting surface of a definite object, like the side of a box, is limited in extent or size. Abstract planes are usually considered to be indefinite in extent. However, for the purpose of solving problems, all planes and all straight lines may be considered to be indefinite in extent, even though they may really have a definite size. A plane or a line, even though it is on a definite object, may be extended beyond the limits of the object if this extension makes the solution easier. The position of the plane or the line in space is not altered by such extension.

A *profile plane* is a plane which shows as an edge in both plan and front elevation views. It is parallel to the image planes for the side elevation views.

12.2. A Plane as a Line or an Edge. View C.

In order to furnish a very convincing proof for a very important statement which is next to be presented, it is suggested that the reader perform the following simple exercise.

Place your lead pencil so that it lies flat against your triangle, and hold them both up in any position in front of you keeping their relative position fixed. Now occupy a position such that

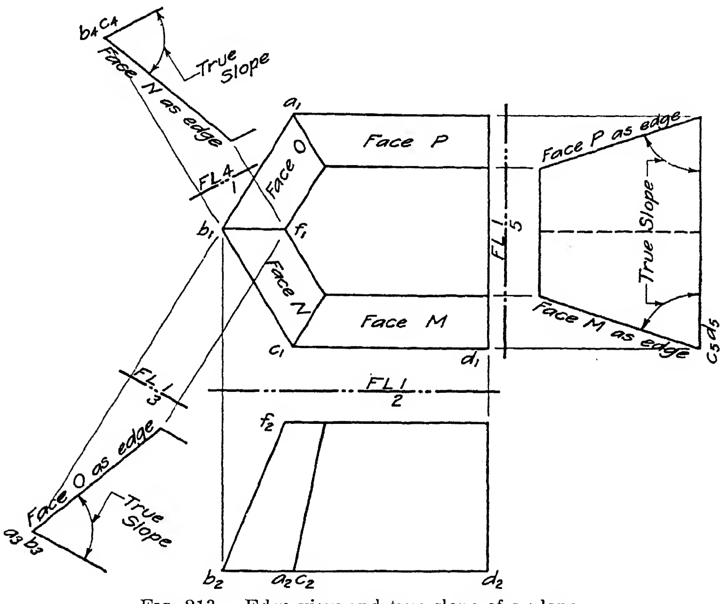


Fig. 213.—Edge view and true slope of a plane.

you can see the pencil appearing as a point and notice that in this view the entire triangle shows as a single straight line. Then try the pencil in some other position on the triangle and see whether the same thing is true again. The triangle represents a plane, and the pencil represents any line lying in that plane. Regardless of where this line lies in the plane, when you see the line as a point the plane appears as a single line or an edge.

This simple illustration of a visual nature gives the next important rule.

Rule 5. Any plane appears as a straight line, or an edge, in the view in which any line in that plane appears as a point.

If it is desired to see the edge view of a plane, any line on that plane may be selected and a view drawn which shows this line as a point, as in Section 9.2. The plane shows as an edge in this new view. This is the general method. However, this method may be shortened by using judgment in selecting the line to see as a point, as illustrated in the following cases.

Case 1. When some line on the plane shows in its true length in one of the given views, that line should be selected to show as a point.

The pier shown in Fig. 213 has four faces (M, N, O, and P) which are inclined planes. The top and bottom of the pier are level planes. The line AB on the face O is level and therefore it shows in its true length in the plan view. An elevation view 3, looking parallel to a_1b_1, shows the line AB as a point. If other points on this face are located in this view, the face will be found to show as an edge, according to Rule 5. In the same way an elevation view 4, looking parallel to the level line BC on the face N, will show that face as an edge. Also an elevation view 5, looking parallel to the line CD on the face M, will show that face as an edge. Also, since the line BF shows in its true length in the front elevation, an inclined view taken from the front elevation and looking parallel to the line BF will show both the face N and the face O as edges.

Case 2. When no line on the plane shows in its true length in either of the given views, draw a new line on the plane that will satisfy this condition. A level line is the best one to select. The prism shown in Fig. 214 has a sloping top plane RSM. No line on this plane shows in its true length in either of the given views. A new line MN, which is level, is drawn on the plane in the front elevation and projected to the plan, where it shows in its true length. The new elevation, view 3, is then drawn to show the line MN as a point. The points M, R, and S are all located in this view by Rule 1 and, after they are located, they are found to lie in a straight line. This gives the edge view of the plane RSM as it should according to Rule 5. The frontal line RV could have been selected instead of the level line MN. However, it is advisable to use the level line in most cases.

13.2. Additional Facts about Edge Views.

A very convincing check of the edge view in the preceding problem will be furnished if the student will actually draw several elevation views of the prism of Fig. 214. He will soon discover that a thousand auxiliary elevation views could be drawn, and that in only two of these views would the plane *RSM* appear as an edge. These two views would be those for which the lines of sight were in the direction of the arrows numbered 1.

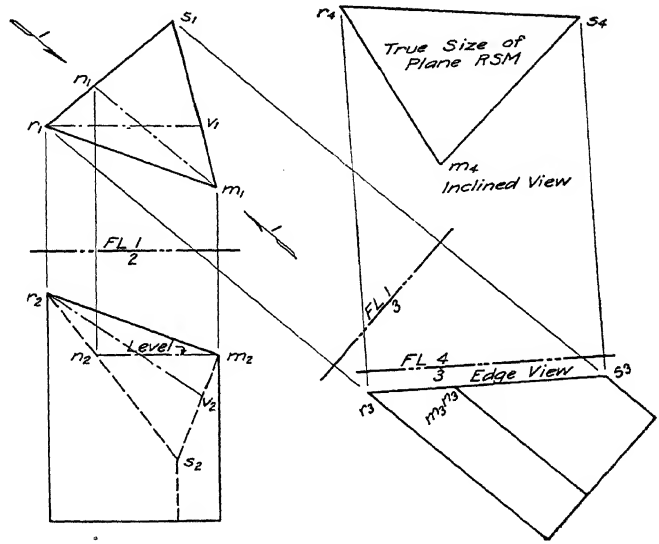


FIG. 214.—True size of any oblique plane.

In like manner, just as many views could be drawn from the front elevation. The only ones that will show the sloping plane *RSM* as an edge are the two inclined views whose lines of sight are parallel to the frontal line *RV* on the plane, because a frontal line always shows in its true length in the front elevation.

Attention is called to the fact that, if the plane *RSM* shows in its true size in the inclined view 4, it must lie parallel to the image plane for this view. Therefore all views taken from this inclined view 4 will show the plane *RSM* as an edge and parallel to the folding line, just as it is in the auxiliary elevation view 3. This fact will be useful in analyzing later problems.

14.2. Practice Problems.

See Chapter VIII, Group 8.

15.2. True Slope of a Plane.

The angle of slope of a plane is the angle the plane makes with a horizontal plane. It may be measured in degrees or by the tangent of the angle, exactly as the slope of a line may be measured.

Rule 6. The true angle of slope of a plane can be seen only in an elevation view which shows the plane as an edge.

The horizontal plane must also appear as an edge. This is the reason why the elevation view is specified in Rule 6, *for it is the only view that can show a level plane as an edge.* In Fig. 213, again, the true angle of slope of each plane is seen in the auxiliary elevation which shows that plane as an edge. It should be noticed that in each case the angle of slope is the angle between the inclined plane and a level plane.

Caution. An inclined view may show a plane as an edge, but it can NEVER *show its true slope,* since it cannot show a level plane as an edge.

Among mining engineers the true slope of a vein of ore is called the "dip." See Section 38.3.

16.2. Practice Problems.

See Chapter VIII, Group 10.

17.2. A Plane in Its True Size. View D.

A plane lying parallel to an image plane, or at right angles to the lines of sight, for any view, shows in its true size and shape in that view. It should be obvious that, in order to see a plane in its true size, one must look with lines of sight at right angles to the plane.

A level plane shows in its true size in the plan view. A vertical plane shows in its true size in the elevation view whose lines of sight are perpendicular to it. But in order to look at right angles to an inclined plane the lines of sight must also be inclined, giving an inclined view.

From the plan and front elevation views of the prism in Fig. 214, it would be very difficult to tell, from those two views alone, where to go to look at right angles to the face *RSM*. But in the auxiliary elevation showing this face as an edge it is apparent that the lines of sight must be at right angles to the plane where

it appears as an edge, in order to be at right angles to the plane. This idea is also stated in the form of a rule, because it will be used over and over again.

Rule 7. A plane must always appear as an edge before a view can be drawn showing it in its true size.

Figure 214 shows how the true size of the face *RSM* is found. The auxiliary elevation view 3 is first drawn, as explained in Section 12.2, in order to show the face *RSM* as an edge. The

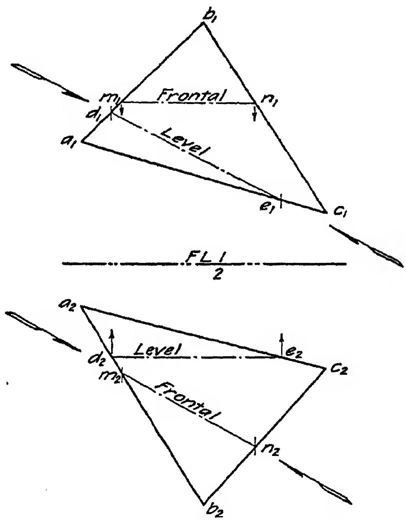


Fig. 215.—Level and frontal lines.

true size of the face *RSM* is then seen in the inclined view 4 by looking at right angles to this face where it appears as an edge. This inclined view shows all the lines on this face in their true lengths and all the angles on this face in their true sizes. This face may now be completely dimensioned in view 4, and any desired measurement may now be made from this view.

18.2. Practice Problems.

See Chapter VIII, Group 11.

19.2. Summary.

Attention is called to the close relationship that exists between these four fundamental views which have just been

explained. View *B* cannot be obtained without having view *A* first. View *C* cannot be drawn without having views *A* and *B* first. View *D* cannot be drawn without having views *A*, *B*, and *C* first. In other words, if it is desired to draw a view showing the true size of a plane, a view must first be drawn which shows the plane as an edge. In order to obtain this edge view of the plane, some line on the plane will have to show as a point in some view. Before a view can be drawn showing a line as a point, there must be a view showing that line in its true length. This last-named view need not always be an extra view, if judgment is used in selecting a line on the plane which already shows in its true length in one of the views already given. The solution is usually simplified by the use of a level or a frontal line on a plane, for these lines show in their true lengths in the given plan and front elevation views, respectively. In Fig. 215 any line on the plane parallel to the line *MN* is a frontal line and will show in its true length in the front elevation. Also any line on the plane parallel to the line *DE* is a level line and will show in its true length in the plan. The four views having lines of sight parallel to the four indicated arrows will all show the plane as an edge but only the two elevation views will show the slope of the plane.

CHAPTER III

POINT, LINE, AND PLANE PROBLEMS

1.3. Introduction.

The common conception of descriptive geometry is that it deals only with abstract lines and planes. This may be correct from the standpoint of a mathematician. But descriptive geometry is an engineering subject and a valuable engineering tool. It was invented by an engineer, Gaspard Monge, for the express purpose of simplifying the solution of structural problems upon which he was working for the French government. Therefore an engineer does not think of lines and planes as being something abstract. He sees them as something real, concrete, and as a vital part of the object he is visualizing. His ability to deal with lines and planes determines his ability to draw correctly all the necessary views of a structure. Sections 12.2 and 17.2 have shown that a real practical object may be composed of just lines and planes and that a logical analysis of these lines and planes may furnish the very key to the desired solution. Most problems may be analyzed and solved by dealing simply with their elementary component parts, namely, the line and the plane.

The purpose of this chapter is to introduce several line and plane problems which frequently occur in practice and which all engineers should know how to solve. These problems may be considered descriptive geometry, but they are just advanced orthographic drawing. Their solution requires knowledge of the relationships between orthographic views and of the method for obtaining any desired view. It also requires the constant use of the seven rules, the fundamental principles, and the four fundamental views as explained in the previous chapters. The same method of logical thinking and three-dimensional visualization will apply, as well as the same notation. In fact, the author is most anxious for the student to realize that the problems to be introduced are all to be solved by the same orthographic methods that were explained in Chapters I and II.

2.3. Theorems.

A theorem is a general statement of a truth, which is capable of being proved. The following theorems are given, mostly without proof, for the purpose of familiarizing the reader with certain truths regarding lines and planes. Not only will their study give considerable practice in visualizing lines and planes in space, but the knowledge of the truths which are set forth will greatly aid in clear analysis of the method to use in solving a

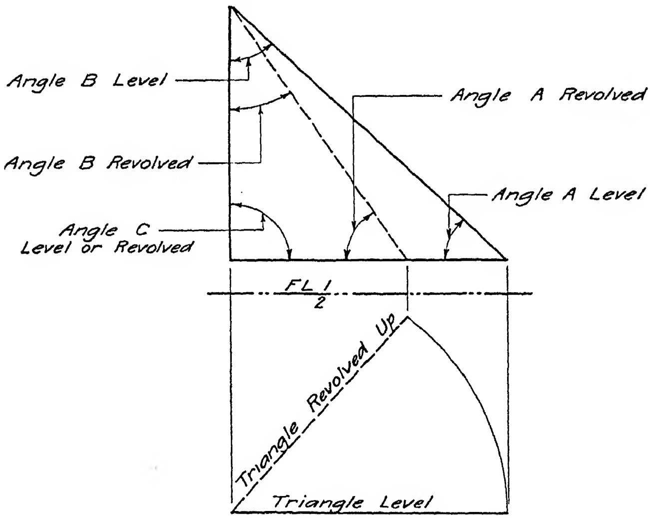


FIG. 301.—Showing how angles may project.

problem. It is therefore strongly recommended that they be thoroughly studied until they have become self-evident truths.

Theorem **1.** *If two lines are parallel to each other in space, they will appear parallel to each other (or as points) in all orthographic views.*

Theorem **2.** *If two lines appear parallel to each other in one view, they are not necessarily parallel to each other in space. If two lines appear parallel to each other in two views, the lines are parallel to each other in space unless they happen to lie parallel to some plane, such as a profile plane, which is parallel to all the lines of sight for the given views.*

Theorem **3.** *If two intersecting lines are at right angles to each other in space, they will appear at right angles to each other in*

any view which shows the true length of either line. (The one exception occurs when the other line shows as a point.)

This theorem may easily be illustrated by laying a 45-degree triangle flat on the desk and then revolving it up about one leg, *BC*, which remains on the desk, until it appears as shown by the dash line in Fig. 301. The angle at *C* is the right angle on the triangle, and it still appears as a right angle in the revolved position. In the revolved position one of its legs shows in its true length, but the other leg does not show in its true length. However, the angle still projects as a right angle.

Theorem **4.** *If two intersecting lines make any angle with each other except a right angle, the view which shows the true size of this angle must show the true length of both lines.*

Theorem **5.** *Any plane angle may project larger or smaller than its true size.*

In Fig. 301, again, the angles *A* and *B* are each 45 degrees, in their true size. The revolved position of the triangle shows the angle *A* appearing larger than its true size and the angle *B* appearing smaller.

Theorem **6.** *An infinite number of lines on a plane may pass through the same point on that plane. Only one of these lines can have the same slope as the plane, and that is the one which makes a right angle with a level line on the plane.*

Theorem **7.** *No line on a plane can have a greater slope than the plane itself has. The slopes of all lines lying on a plane vary from zero to a maximum value equal to the slope of the plane.*

Theorems 6 and 7 can be easily demonstrated by placing a pencil in several positions on a sloping plane and observing the different slopes it can possibly have.

Theorem **8.** *If two lines lie on the same plane either they must be parallel or they must intersect each other.*

Theorem **9.** *Two lines that lie in two different intersecting planes cannot be parallel to each other unless they are both parallel to the line of intersection of the two planes.*

3.3. To Project a Point on a Line from One View to Another View When the Line Is Parallel to a Profile Plane.

ANALYSIS.

The location of the point on the line would be known in one view, but it would be impossible to project it directly to the other

view. Accordingly any new view of the line must be drawn and the point located on the line in this view. This new view may be a side elevation, any auxiliary elevation or any inclined view. The point, being located in the two views, may easily be located in all other views by projection.

Explanation (see Fig. 302).

The plan and the front elevation of the line AB are given. The location of the point X on the line AB is known only in

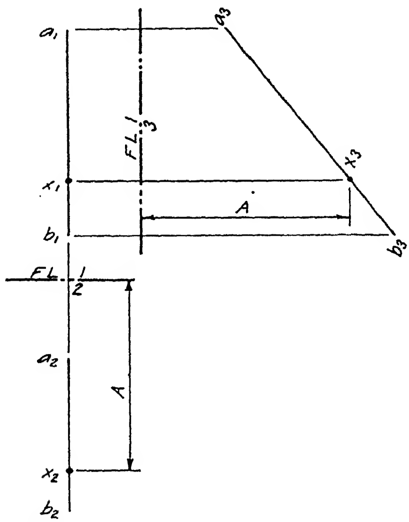


Fig. 302.—Locating a point on a line in other views.

the front elevation. It is evident that the point X could not be projected directly to the plan. A new view 3, of the line AB is drawn. The point X is located on the line AB in this view by the use of Rule 1, as it must be A distance below the folding line. It is then projected directly to the plan from this view. Also any auxiliary elevation could be drawn from the plan, and the point X located in the same way. Also any inclined view could be drawn from the front elevation, and the point X located on the line AB in this view by projection and again in the plan by measurement according to Rule 2.

4.3. To Project a Point Which Is on a Given Plane, Having Only One View of the Point Given.

ANALYSIS.

The given plane would be represented by either parallel or intersecting straight lines. If the given point is on one of the given lines of the plane in one view, it will be on that line in every view; it can be located on that line in any view by simple projection. If the given point is not on any of the given lines of the plane, an auxiliary line is drawn on the plane so that it contains the given point and intersects the given lines of the plane. This auxiliary line is then projected to any other view, and the given point is located on that line in that view.

Also an edge view of the plane may be drawn and the given point located on that edge view either by elevation or by projection.

5.3. Practice Problems.

See Chapter VIII, Group 12.

6.3. To Draw the Plan and Front Elevation of a Line, Having Given Its Bearing, Its True Slope, and Its True Length.

Note. In problems involving bearing, north is always taken at the top of the paper, and the front elevation is always assumed to be looking due north unless otherwise specified.

ANALYSIS.

If the bearing of a line is given, the direction of the plan view is absolutely fixed and the plan view may be drawn indefinite in length. With the plan view fixed, a new elevation with lines of sight at right angles to the line will show its true length and its true slope. In this new elevation view the line may be drawn in, with its true slope and its true length. Both ends of the line are now determined, and may be projected back to the plan or to any other view.

Explanation (see Fig. 303).

The bearing of the line AB is given as N 30°E from the point A. The true slope is known to be $-35°$ and the true length is known to be 3 in. The plan view of the line AB is drawn first, disregarding its length. View 2 is drawn next with lines of sight

at right angles to the line AB. In this view the line AB shows its true slope and its true length as given. This determines both ends of the line, for the point A may be placed at any elevation. Both ends of the line are then projected back to

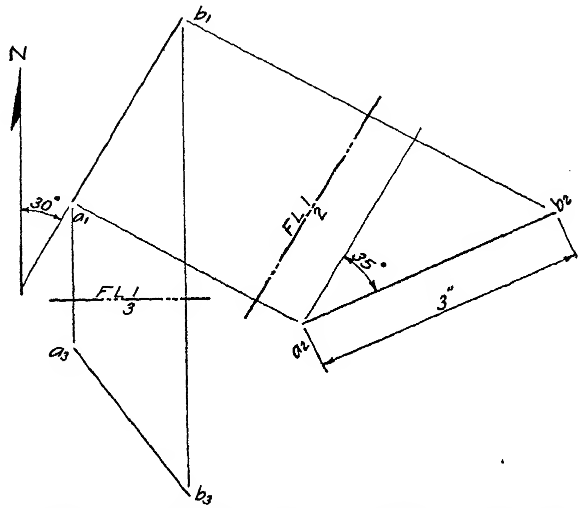


Fig. 303.—A line having a definite bearing, slope, and length.

the plan and from there to the front elevation, by Rule 1. It is now possible to locate the line in any other view desired.

7.3. Practice Problems.

See Chapter VIII, Group 13.

8.3. To Find the Perpendicular Distance from Any Point to Any Line.

First Method, or Line Method.

ANALYSIS.

In the view which shows the given line as a point the required perpendicular distance shows in its true length, because it lies at right angles to the lines of sight for this view. The foregoing statement may be easily visualized by holding up a 90-degree triangle. The view which shows one leg of a 90-degree angle as a point will also show the other leg of that angle in its

true length. The solution, then, consists in simply obtaining a view of the given point and the given line which will show the given line as a point (see Fundamental View *B*).

Explanation (see Fig. 304).

The line *AB* and the point *X* are given in the plan and front elevation views. A new elevation view, 3, is drawn to show the line in its true length. The inclined view 4 is then drawn to show the line *AB* as a point, and the point *X* is shown in this same view.

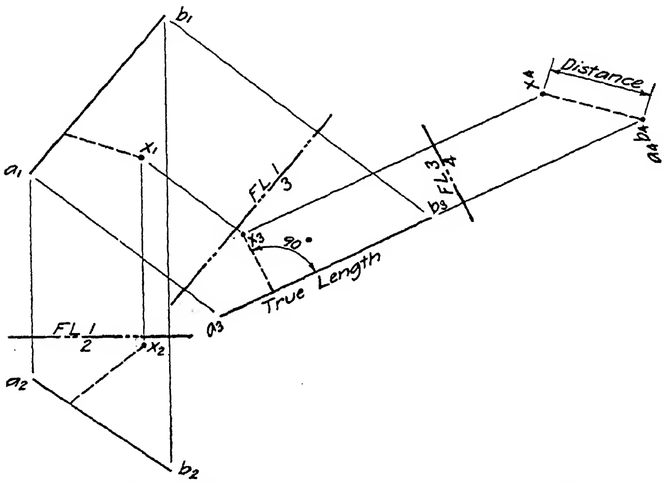


FIG. 304.—Distance from a point to a line. Line method.

The distance from x_4 to a_4b_4 is the true length of the perpendicular distance from the point to the line. It might be required to find where this perpendicular would meet the line *AB*. Since the perpendicular is at right angles to the line *AB* in space, it projects at right angles to the line in view 3, because the line shows in its true length in this view (see Theorem 3). The perpendicular is shown as a dashed line in all views.

Second Method, or Plane Method.

ANALYSIS.

A plane may be passed through any three points. The three points *A*, *B*, and *X* are considered to locate a plane. The true

size of this plane is determined exactly as in Section 17.2. In this view showing the true size of the plane, the perpendicular distance shows in its true length. The drawing is not shown here, for it would exactly reproduce the method followed in Fig. 214.

9.3. Practice Problems.

See Chapter VIII, Group 14.

10.3. To Draw a Plane Which Contains One Given Line and is Parallel to Another Given Line.

ANALYSIS.

By geometry, if a line is parallel to any line on a plane, it is parallel to the plane. A third or auxiliary line is drawn which

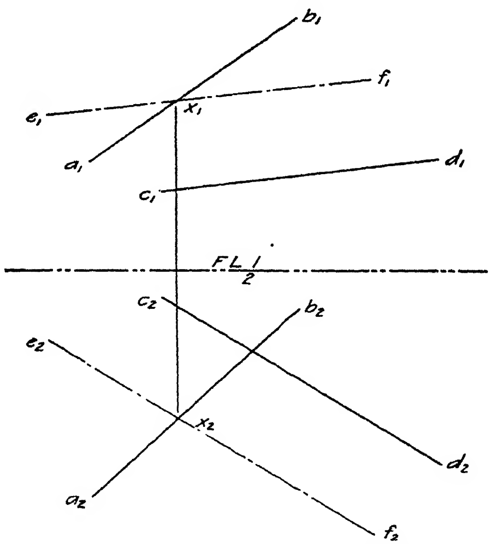


Fig. 305.—Plane parallel to one line and containing another line.

intersects one of the given lines and is parallel to the other given line. The two lines which now intersect must determine a plane. The other line must be parallel to this plane because it is parallel to the auxiliary line which is on the plane. The required plane is represented in both views by the two intersecting lines.

Explanation (see Fig. 305).

The two nonintersecting and nonparallel lines, AB and CD, are given in both views. Through any definite point on the line AB, such as X, the line EF is drawn so as to be parallel to the line CD in space. Since the lines AB and EF intersect at the point X, these two intersecting lines determine the required plane. The line AB is on this plane, and the line CD is parallel to this plane because it is parallel to the line EF which is on the plane.

In order to check this solution, a new view may be drawn showing the plane $ABEF$ as an edge and showing the line CD in this same view. The line CD should check parallel to the edge view of the plane.

A plane could also have been drawn so as to contain the line CD and be parallel to the line AB.

11.3. Practice Problems.

See Chapter VIII, Group 15.

12.3. To Determine the Shortest Distance between Any Two Nonintersecting, Nonparallel Lines.

First Method, or Line Method.

ANALYSIS.

The shortest distance from any point to any line is the perpendicular distance from the point to the line. The shortest distance between two lines will have to be measured perpendicular to both lines. At one, and only one, definitely fixed position in space is it possible to have a line perpendicular to two other lines as specified in this problem. This statement may be visualized by holding up two pencils so that they are nonparallel and nonintersecting, and by trying to see where this shortest distance would be measured. This common perpendicular to both lines appears in its true length in the view which shows either one of the lines as a point. Also in this same view, this perpendicular, being in its true length, projects at right angles to the other line that is not in its true length because the two lines are at right angles to each other in space (see Theorem 3). The solution, then, consists in drawing a new view of both lines which shows one line as a point and in drawing the shortest line through this point and perpendicular to the other line.

Explanation (see Fig. 306).

The two nonintersecting and nonparallel lines, AB and CD, are given in the plan and front elevation views. A new view, 3, is drawn to show the true length of the line AB. The line CD is also shown in this view. View 4 is drawn to show the line AB as a point. The line CD is also shown in this view, but it does not show in its true length. However, the common perpendicular to the two lines is in its true length in this view and may therefore be drawn at right angles to the line CD. This determines the

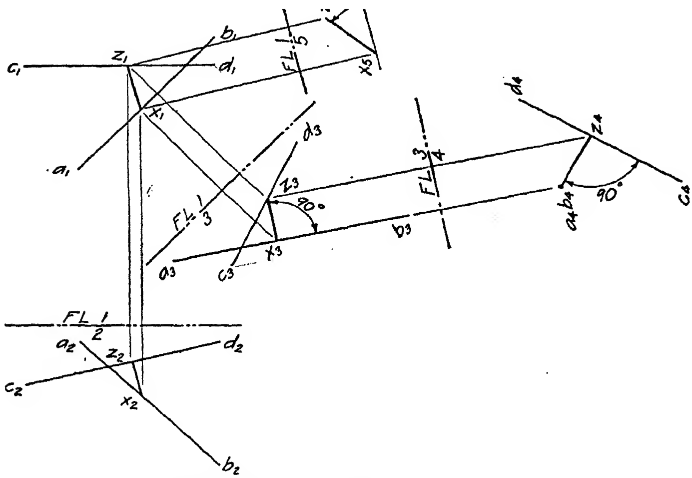


Fig. 306.—Shortest distance between two lines. Line method.

point Z. The true length of the shortest distance may now be measured in view 4.

If it is desired to determine the bearing and the true slope of this common perpendicular, it must be projected back to the plan view. The point Z is projected to the line CD in view 3, and the perpendicular is then drawn from z_3 at right angles to the line AB, which is in its true length in this view. The intersection determines the point X. Both points, X and Z, may now be projected to any desired view. The bearing of the line XZ is read in the plan. To determine its true slope, a new elevation view, 5, must be drawn showing the line in its true length. The true length as found in view 5 should check with the true length found in view 4.

Second Method, or Plane Method.

ANALYSIS.

If a plane is drawn containing one of the lines and parallel to the other, an elevation view showing this plane as an edge will show the other line to be parallel to the plane. It will also show the true length and true slope of the shortest or perpendicular distance between the two lines. This is true because this perpendicular distance would have to be perpendicular to the plane assumed, and, since the plane shows as an edge, a line at right angles to the plane will have to show in its true length.

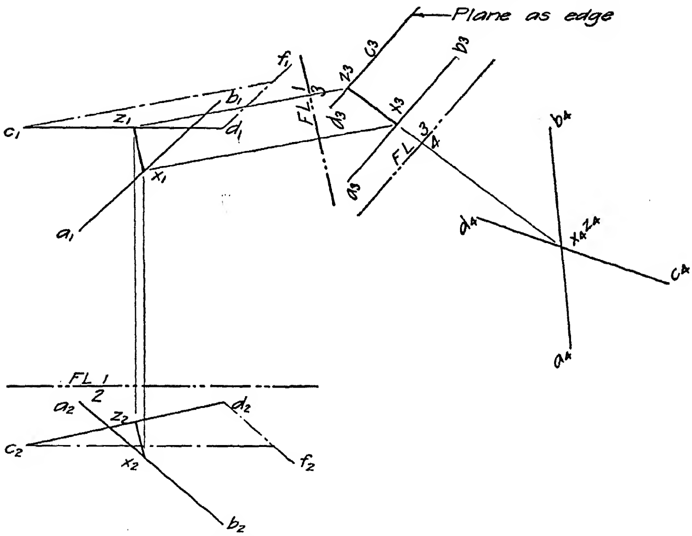


FIG. 307.—Shortest distance between two lines. Plane method.

However, the exact location of this perpendicular would not yet be determined. A new view of the line and the plane taken at right angles to the plane will show both the given lines in their true lengths. The common perpendicular to these two lines will then appear as the point where the two lines appear to intersect.

Explanation (see Fig. 307).

The two nonintersecting nonparallel lines, AB and CD, are given in the plan and front elevation views. By the method of

Section 10.3, the plane CDF is so drawn that it contains the line CD and is parallel to the line AB. View 3 is drawn, showing this plane as an edge and also showing the line AB to be parallel to the plane. Another view, 4, is drawn showing the plane in its true size and both the given lines in their true length. In view 4, where the two lines appear to intersect, the common perpendicular to these two lines appears as a point at x_4z_4. This perpendicular may now be projected back to all other views; its true length and true slope are both shown in view 3.

The engineering application of this problem is made in determining the shortest distance between two tunnels, the location of 90-degree fittings for connecting two pipes, or the shortest distance between two electric wires.

13.3. Practice Problems.

See Chapter VIII, Groups 16 and 17.

14.3. To Determine the Shortest Level Line Connecting Two Nonintersecting, Nonparallel Lines.

ANALYSIS.

A plane is drawn containing one of the lines and parallel to the other line. In an auxiliary elevation view showing the plane as an edge the other line shows parallel to the plane. An infinite number of level lines, all lying at different elevations, could connect these two given lines but they all must lie parallel to the folding line in this auxiliary elevation. The exact elevation at which the shortest one lies is still unknown. Since no level connection can be shorter than the actual level distance between these two lines as it appears in this auxiliary elevation, this apparent distance must be the shortest level distance in its true length. One more elevation, related to the auxiliary elevation, is drawn and the shortest level distance shows as a point where the two given lines appear to intersect, because it was in its true length in the preceding view.

Explanation (see Fig. 308).

The two nonintersecting nonparallel lines, AB and CD, are given in the plan and front elevation. By the method of Section 10.3, the plane CDF is drawn containing the line CD and parallel to the line AB. View 3 shows this plane as an edge and line AB

parallel to it. In view 3 the shortest level connection is in its true length and lies parallel to the folding line and in view 4 it appears as a point at x_4z_4 where the two given lines appear to

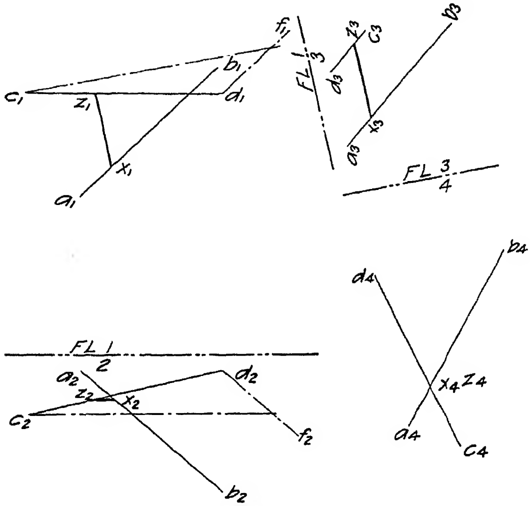


FIG. 308.—Shortest level distance between two lines.

intersect. Its position is now definitely determined and it is located in all other views by projection.

15.3. Practice Problems.

See Chapter VIII, Group 18.

16.3. To Find the Line of Intersection of Two Planes When One Plane Appears as an Edge in One of the Given Views.

ANALYSIS.

By solid geometry, two planes must intersect in a straight line, every point of which lies on both planes. Also, any two points determine the direction of a straight line. In the view in which one of the planes appears as an edge the points will be apparent where any two lines on the other plane pass through, or pierce, the edge view. Those two piercing points are on both planes, and therefore they are on the line of intersection of the two.

They will be sufficient to determine the line of intersection in all views.

Explanation (see Fig. 309).

The two given planes are the plane ABC and the plane XY, which shows as an edge in the plan view. The lines AB and AC are both on the plane ABC. In the plan view it is apparent that the lines AB and AC pass through the plane XY at the points

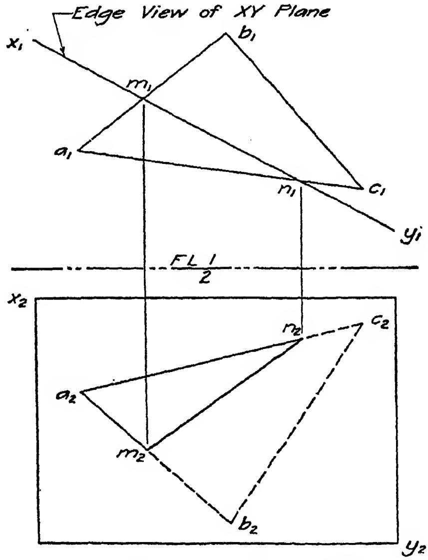


Fig. 309.—Plane cut by a plane appearing as an edge.

M and N, respectively. Therefore the points M and N must lie on both the given planes and therefore on the line of intersection of the two. In any other view than the plan, M and N are located by projection, and in all views they determine the direction of the required line of intersection, regardless of its length. The portion AMN of the plane ABC is in front of, and the portion $MNCB$ is behind, the plane XY.

17.3. To Find Where a Line Pierces an Oblique Plane.

ANALYSIS.

A new view is drawn showing the plane as an edge. The line is shown in this same view. The exact point at which the line

pierces the plane will be apparent in this view, just as it was in Section 16.3. The new view to be drawn may be either an elevation edge view or an inclined edge view.

18.3. To Find Where a Line Pierces an Oblique Plane, Using Only the Two Given Views.

First Method: By Vertical Projecting Plane.

ANALYSIS.

The vertical projecting plane for the given line is the plane which contains all the vertical lines of sight for the line, or it

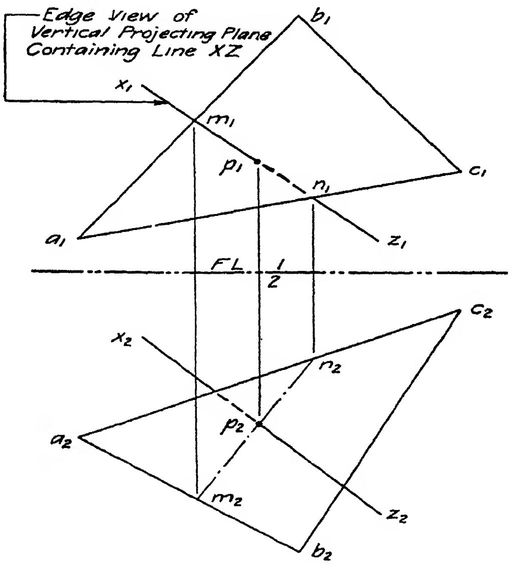


FIG. 310.—Line piercing an oblique plane. Vertical projecting-plane method.

is the plane which actually projects the line on to the plan image plane. This vertical projecting plane shows as an edge in the plan view, and of course, contains the given line. By the method of Section 16.3, the intersection of this vertical projecting plane with the given plane is determined. The given line and this line of intersection both lie in the vertical projecting plane, and therefore they must either intersect or be parallel. This relationship cannot be seen in the plan because the entire plane appears as an edge. However, it is apparent in the other

given view. If the two lines intersect, the point of intersection is on both the given line and the given plane and is therefore the point at which the given line pierces the given plane.

Explanation (see Fig. 310).

The line XZ and the oblique plane ABC are given. The vertical projecting plane of the line XZ appears as an edge in the plan as x_1z_1. The intersection of this vertical plane with the plane ABC is found to be the line MN, which is shown in both views. The lines MN and XZ both lie in the vertical projecting plane, and in the front elevation these lines are seen to intersect at the point P. This point is on the line MN, every point of which is on the plane ABC, and therefore it is also on the plane ABC. But it is also on the line XZ. Therefore it is the required point where the line XZ pierces the plane ABC. If the front elevation had shown the line XZ to be parallel to the line MN, that would have been proof that the line XZ was parallel to the plane, because it was parallel to a line MN on the plane. It is evident that there could be no piercing point in that case.

Second Method: By Front Projecting Plane.

ANALYSIS.

The front projecting plane of the line is the plane which projects the line upon the front image plane. The line of intersection of the front projecting plane with the given plane is determined, and the method from here is exactly the same as the first method in Section 18.3. However, in this case the piercing point is apparent in the plan view instead of in the front elevation.

Explanation (see Fig. 311).

The line XZ and the oblique plane ABC are given. The front projecting plane containing the line XZ appears as an edge at x_2z_2. The intersection of this plane with the plane ABC is found to be the line RS. In the plan it is apparent that the lines XZ and RS intersect at the point P. This point is therefore on both the given line and the given plane, and is the point required.

The methods of Sections 17.3 and 18.3 furnish three absolutely independent ways for determining where a line pierces an oblique

plane. All three methods should determine exactly the same point, and if they are all used on the same problem they will furnish an excellent check on the accuracy of the drafting work. In most engineering settings it is easier to solve the problem in just the two views given. But in some cases it may be easier to

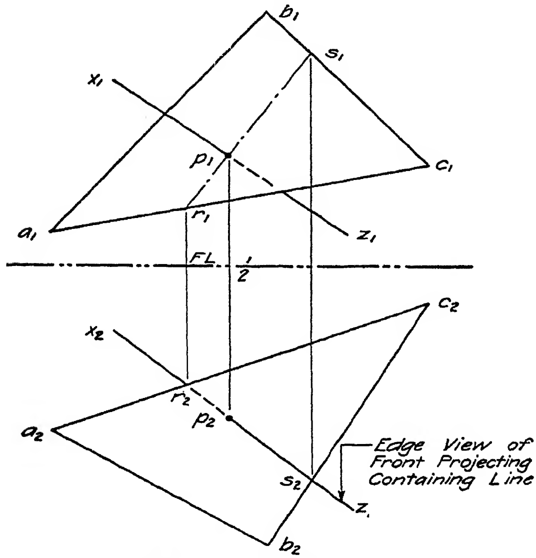


FIG. 311.—Line piercing an oblique plane. Front projecting-plane method.

draw the new edge view. Judgment must always be used by the draftsman in selecting the best method to use for each job.

19.3. Practice Problems.

See Chapter VIII, Group 19.

20.3. To Find the Line of Intersection of Any Two Oblique Planes.

ANALYSIS.

A new view, elevation or inclined, may be drawn showing one of the planes as an edge, and including the other plane. This new view will indicate the points at which any two lines on the second plane pierce the plane which appears as an edge. The line of intersection is determined by these two piercing

points. The new view reduces this problem to the same form as that in Section 16.3.

21.3. To Find the Line of Intersection of Any Two Oblique Planes, Using Only the Two Given Views.

First Method: Line Method.

ANALYSIS.

Any line on one of the planes may be selected, and the point at which it pierces the other plane may be determined by the methods of Section 18.3. This point lies on both planes. The same process may be repeated for some different line, which will determine a second point on both planes. These two points determine the entire line of intersection of the two given planes. However, if two more points were to be determined in the same way, all four points should prove to be in a straight line.

Second Method: Auxiliary-plane Method.

ANALYSIS.

A third or auxiliary plane, sometimes called a cutting plane, may be drawn in any position in either view, if it appears in one view as an edge. The intersection of this plane with each of the given planes may be found by the method of Section 16.3. These two lines of intersection both lie on the auxiliary plane, and therefore they must either intersect or be parallel. In case they intersect, the common point on the two lines of intersection will be apparent in one view and may be projected to the other. This point is on both the given planes, and therefore is on their line of intersection. The process is repeated by taking another auxiliary plane, one point on the line of intersection being determined by each plane. Two points thus determined are sufficient to establish the required line of intersection, although a third point should always be determined for a check.

Explanation (see Fig. 312).

The planes ABC and YXZ are the two given oblique planes. The first auxiliary plane to be assumed is a vertical plane which is found to intersect the given planes in the lines HK and RS as shown. The front elevation shows these two lines intersecting at the point M, which is projected back to the plan. This point

is on both the given planes, or the planes extended, and is therefore on their line of intersection.

A second auxiliary plane is taken appearing as an edge in the front elevation, and similarly the point N is found to lie on both the given planes. The points M and N are sufficient to determine the direction of the entire line of intersection in any view. It is safer to determine at least three points on the line of intersection, and to see whether they check in a straight line in all views. Any other cutting plane may be drawn at any angle in

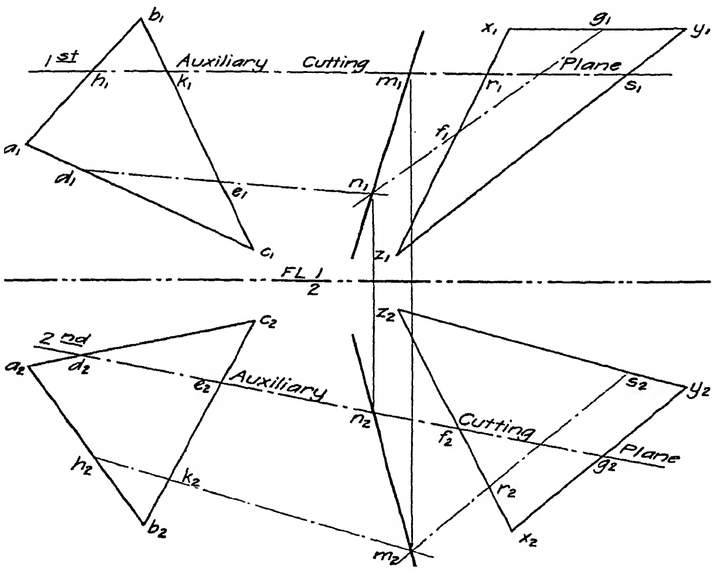


FIG. 312.—Intersection of two planes. Auxiliary-plane method.

either view, as long as it appears as an edge in one of the given views. Attention has already been called to the fact that, for purposes of solution, planes may be considered to be indefinite in extent. In this case the points M and N are not inside the limits of the planes as given, but they are on both planes extended. The line of intersection of these two planes cannot possibly be in any other position, regardless of how large the planes are.

If the lines HK and RS should happen to be parallel instead of intersecting, this fact would prove these two lines were parallel to the line of intersection of the given planes (see Theorem 9).

While the direction of the required line of intersection would then be known, its position would still be unknown. In such a case the vertical plane should be assumed at some different angle with the front image plane, and the problem could be completed. If this different vertical plane still intersects the two given planes in two parallel lines, the given planes themselves must be parallel.

Three independent methods of finding the line of intersection of two planes have been explained in Sections 20.3 and 21.3. Each method has its advantages and is more easily applied to some problems than the other two. The draftsman should be familiar with all three methods.

22.3. Practice Problems.

See Chapter VIII, Group 20.

23.3. To Find the True Size of the Dihedral Angle between Any Two Intersecting Planes.

ANALYSIS.

The dihedral angle formed by two intersecting planes is measured by the plane angle cut out of the given planes by a plane which is perpendicular to their line of intersection. An auxiliary view which shows this line of intersection as a point also shows the true size of the plane angle that measures the dihedral angle. Therefore the dihedral angle shows in its true size in the view which shows the line of intersection of the two planes as a point. Since this line of intersection is on both planes, both of these planes will appear as edges in the view in which the line of intersection appears as a point. It remains only to locate any one point on each plane in this new view, beside the line of intersection, in order to locate the edge view of each plane.

Explanation (see Fig. 313).

It is desired to find the true size of the dihedral angle at the valley AB between the two roofs $ABED$ and ABC, shown in the plan and front elevation. A new elevation view, 3, is first drawn showing the line of intersection AB in its true length. The points C and D are chosen at random, one on each roof, and they are located in view 3 also. View 4 is drawn showing the line AB as a point, and showing also the points C and D. Since both planes

must appear as edges in view 4, these planes may both be drawn in to the line of intersection as a point, since one other point is located on each plane. The dihedral angle is now shown in its true size in view 4, where it may be scaled.

To show how practicable this method is, two more views, 5 and 6, are drawn from view 4, at right angles to each plane. Each plane will then appear in its true size. If this valley angle is to be detailed as a steel-connection angle, views 4, 5, and 6 are all that are required to make a complete shop drawing for steel

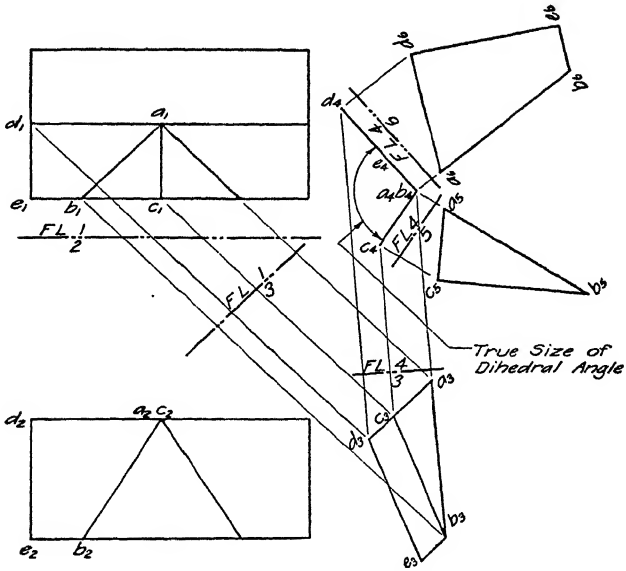


FIG. 313.—Angle between two planes.

fabrication. The scale of the drawing, from view 4 on, might easily be enlarged, but the angles, cuts, and lengths all appear in their true size in the three views shown, and the relationship between these views is the same as for any simple shop drawing. Structural-steel draftsmen utilize this method constantly in determining the angles and cuts for bent-plate connections.

In the problem of Fig. 313 the line of intersection between the two planes was given. In any other case, it is necessary only to determine this line of intersection as in Sections 20.3 and 21.3, and to proceed from there as in the problem above.

24.3. Practice Problems.

See Chapter VIII, Group 21.

25.3. To Draw a Line Perpendicular to a Plane from a Point Not on the Plane.

ANALYSIS.

A new view of the point and the plane should be drawn which will show the plane as an edge. In this view the perpendicular

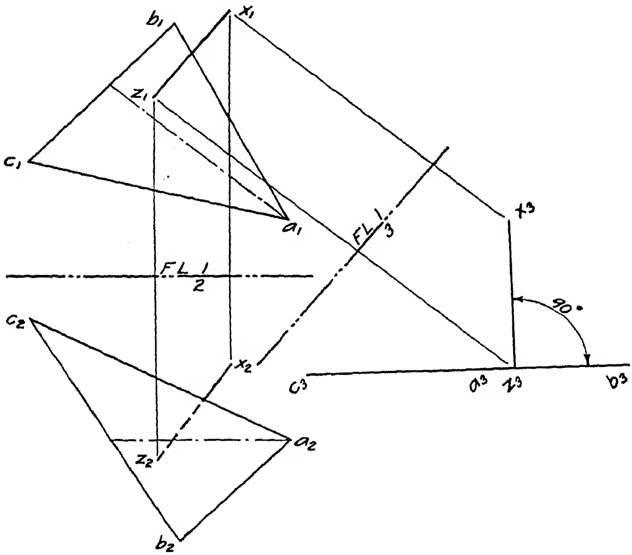


FIG. 314.—Line perpendicular to a plane. Edge-view method.

line from the point to the plane can be drawn at right angles to the edge view of the plane. This perpendicular line appears in its true length in this new view, which information determines the plan view of the line; it is parallel to the folding line or at right angles to the lines of sight for the new view. The point where the perpendicular pierces the plane is also evident in the new view.

Explanation (see Fig. 314).

The plane *ABC* and the point *X* are given in plan and front elevation. View 3 is drawn to show the plane as an edge and

to show the point X. From x_1 the perpendicular line may be drawn at right angles to the plane, and it is seen to pierce the plane at z_3. The line XZ shows in its true length in view 3, and therefore the lines of sight for this view must be at right angles to the line XZ in the plan. This fixes the plan view at x_1z_1. The front elevation of the line XZ is obtained by projection.

26.3. To Draw a Line Perpendicular to a Plane from a Point Not on the Plane, Using Two Views Only.

ANALYSIS.

By solid geometry, if a line is perpendicular to a plane, it is perpendicular to all lines on the plane that intersect the perpendicular. By Theorem 3, if one of these intersecting lines on the plane should show in its true length in some view, the perpendicular to the plane would project at right angles to that line in that view.

Any number of lines on a plane may show in their true length in a given view, but they are all parallel. Then any one of these true-length lines on a plane determines the *direction* of the perpendicular to the plane in that view even though it is not the particular line that actually intersects the perpendicular.

The statements just given may be condensed into the following simple rule:

Rule 8. Direction Principle. In any orthographic view, a line that is perpendicular to an oblique plane in space shows at right angles to any line on the plane that shows in its true length in that view.

Caution. Rule 8 *determines only the direction of the perpendicular to a plane in any view; it does not determine the point where the perpendicular pierces the plane.* This piercing point is determined separately by any of the three methods already explained. It must also be remembered that the direction of a line must be found in two views before the position of the line itself is fixed in space.

Explanation (see Fig. 315).

The plane ABC and the point X are given in the two views. Any level line at random, such as the line AM, is chosen on the plane and is drawn in both views. The perpendicular to the plane from the point X is next drawn in the plan view at right

angles to the line AM which shows in its true length in the plan. Since all level lines on the plane are parallel, any other level line on the plane would have given the same direction to the plan view of the perpendicular. This perpendicular to the plane is drawn indefinite in length because the piercing point is still unknown.

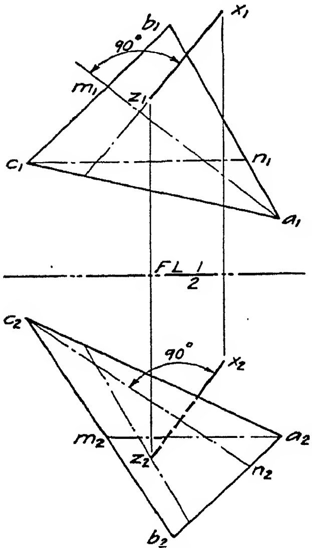


FIG. 315.—Line perpendicular to a plane. Direction principle.

In the same way a random frontal line, such as CN, is drawn on the plane and the perpendicular to the plane is drawn from the point X at right angles to the line CN in the front elevation and indefinite in length. The perpendicular to the plane is now fixed in two views; the point where it pierces the plane is determined by the method of Section 18.3 and is found to be the point Z. The same process would be followed for locating the perpendicular to the plane in any other view. A new elevation view is required if it is desired to show the true length and true slope of this perpendicular distance.

27.3. Practice Problems.

See Chapter VIII, Group 22.

28.3. To Project a Line upon an Oblique Plane.

ANALYSIS.

To project a line upon a plane means to draw a series of lines from the given line *perpendicular to the given plane* and to find where these perpendiculars pierce the plane. Two perpendiculars, one from each end of the line, are sufficient to project a straight line upon a plane. The method of drawing the perpendiculars and finding their piercing points is exactly the same as in Section 26.3. The line connecting the two piercing points is the projection sought.

29.3. Practice Problems.

See Chapter VIII, Group 23.

30.3. To Find the True Size of the Angle a Line Makes with a Plane.

First Method.

ANALYSIS.

The angle a line makes with a plane is the angle between the line and its projection on the plane. This angle lies in a pro-

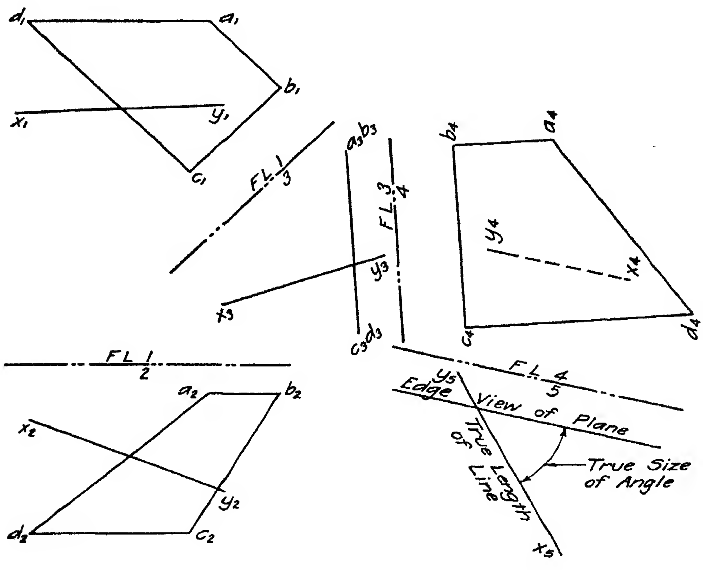


FIG. 316.—Angle between a line and a plane.

jecting plane containing the line and perpendicular to the given plane. In order to show this angle in its true size the *edge view of the plane* and the *true length of the line* must be seen in the same view. First, a view should be drawn showing the plane in its true size and also showing the line. Any view related to this one shows the plane as an edge. A view is then drawn from the view showing the true size of the plane, to show the true length of the line. The desired angle between the line and the plane is seen in this view.

Explanation (see Fig. 316).

The plane $ABCD$ and the line XY are given in plan and front elevation. The true size of this given plane is first found as in Section 17.2. This true size is shown in view 4, together with the line XY. View 5 is then drawn to show the line XY in its true length. In this view the plane appears as an edge parallel to the folding line. This may be checked by measuring several points on the plane according to Rule 2. The true size of the angle between the line XY and the plane may be scaled in view 5.

Since view 4 shows the true size of the plane, the lines of sight for this view must be perpendicular to the plane. Therefore, in view 4, the line x_4y_4 is the edge view of the plane which contains the angle between the line XY and its projection on the plane.

In solving problems by this method it should always be remembered that *the first thing to do is to find the true size of the plane.* This true-size view is the *key view.*

Second Method.

ANALYSIS.

It has already been stated that the angle a line makes with a plane is the angle between the line and its projection on the plane. The line and its projection on the given plane both lie in the projecting plane whose true size may be found by the ordinary methods. This method will give the true size of the angle required with one less view than the first method, but it is harder.

31.3. Practice Problems.

See Chapter VIII, Group 24.

32.3. To Draw Any Required Views of a Plane Figure Which Lies on an Oblique Plane.

ANALYSIS.

A view should be obtained first which shows the true size of the plane. The given plane figure may then be drawn on the plane in its true size, being so placed on the plane as to satisfy whatever conditions were specified. This plane figure may then be projected, point by point, back to all other views, by the ordinary methods of orthographic projection.

33.3. Practice Problems.

See Chapter VIII, Group 25.

34.3. To Draw Any Required Views of a Circle Which Lies on an Oblique Plane.

ANALYSIS.

A circle may be treated like any other plane figure and projected into different views by applying the method of Section 32.3 to a large number of points on the circle. This becomes a cumbersome method for a circle. It is much easier to obtain the views of a circle by locating its major and minor axes, because a circle will project as an ellipse or as a circle in all views except the edge view.

A circle may have any number of lines through its center as diameters. One of these diameters is always a level diameter which shows in its true length in the plan. This level diameter is the major axis of the ellipse in the plan and the minor axis is at right angles to it. A new view may be drawn showing this level diameter as a point; the plane of the circle is an edge in this view. On this edge view the circle may be laid off in its true length, or diameter, and projected back to the plan view to give the length of the minor axis. The two axes having been determined, the ellipse may be drawn in by the card or trammel method (see Appendix, page 221) to complete the plan view.

In exactly the same way the front elevation may be obtained by choosing a diameter of the circle which is a frontal line and proceeding as before. Any other view may be obtained as easily by choosing a diameter of the circle that shows in its true length in that view.

Explanation (see Fig. 317).

The plane ABC and the point X on this plane are given. The point X is the center of a circle having a given diameter and lying on the plane. In the plan view a level diameter is drawn through the point X parallel to any level line on the plane, such as the level line through the point A. This level diameter is laid off in its true length, the actual diameter of the circle, to give the major axis for the ellipse in the plan. View 3 is drawn to show the plane as an edge. The major axis for the plan appears as a point in this view at x_3 which is also the center of the circle. In

view 3 the circle is laid out in its full diameter and then projected back to the plan to give the length of the minor axis. The ellipse is drawn in by the trammel method (see Appendix, page 221).

To obtain the front elevation a frontal diameter is drawn through the point X, parallel to a frontal line on the plane, and is laid off in its true length to give the major axis for the front elevation. The new view 4 is drawn from the front elevation as shown and the procedure is exactly the same as for the plan view.

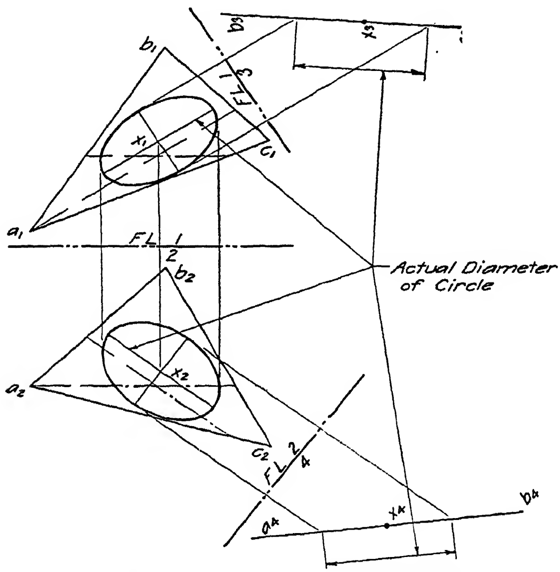


Fig. 317.—A circle lying on an oblique plane.

The method which has just been explained will be found to be the easiest, fastest, and most accurate way of drawing circles on inclined planes. It should be remembered that the major and minor axis for each view are entirely different sets of lines. When the related views of a circle have been completed, each point of the circle should project perfectly between views. The extreme projecting points in related views furnish a quick check.

35.3. Practice Problems.

See Chapter VIII, Group 26.

36.3. To Draw Views of a Solid Object Resting on an Oblique Plane.

ANALYSIS.

The solid object can usually be drawn in its simplest position in the view showing the plane as an edge. Whether it can or not depends upon the object itself. In some cases a view might have to be drawn showing the true size of the plane before the object could be drawn in its correct position. After the object has been

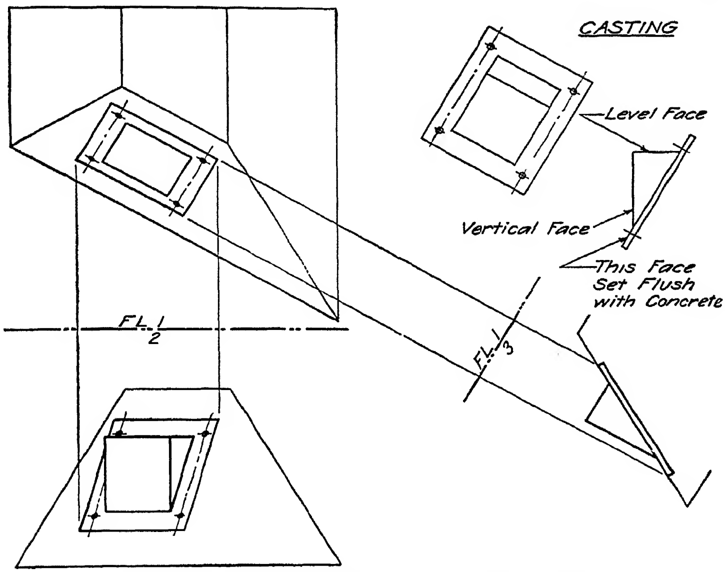


FIG. 318.—Solid object resting on an oblique plane.

placed in the correct position in either the edge or the true-size view of the plane, it may easily be projected to all other views.

Explanation (see Fig. 318).

The plan and front elevations of the end portion of a concrete pier are given. Also two views are given of a casting which is to be placed in the end face of the pier, as specified on the drawing. The new view 3 is first drawn showing the end face of the pier as an edge. The casting is then drawn in position in this view with its proper face set flush with the concrete as specified. The exact location of the casting on the face was not specified

in this problem. A view could be drawn off from view 3 to show the other view of the casting in position. However, in this case such a view is not necessary, for it would be merely a reproduction of one of the given views of the casting. Any necessary measurements may be taken from the given view, if it is drawn to the same scale or from the given dimensions.

37.3. Practice Problems.

See Chapter VIII, Group 27.

38.3. Mining Problems.

The principles which have been explained thus far are quite applicable in the solution of problems encountered by the mining engineer. A vein of ore, within certain limits, may be considered to be just a plane having some thickness and nearly always some incline. The terms commonly used by mining engineers for giving the lay of a vein of ore are the "strike" and the "dip."

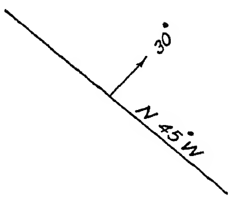


FIG. 319.—Method of indicating strike and dip.

The strike of a vein is the bearing of a level line on the plane of the vein.

The dip of a vein is the true slope of the vein, and is the angle of incline measured below a horizontal plane and at right angles to the strike. Figure 319 shows one conventional way of describing a vein which has a strike of N 45° W and a dip of 30°. The dip angle is intended to be down in the direction shown by the arrow.

To show on a map the exact position of a vein of ore, definite points are located where the vein is visible at the surface of the ground. These points are called "points of outcrop" and they may be located accurately by ordinary surveying methods, both as to elevation and as to position on the map. If no points of outcrop are available, boreholes are sunk until they meet a vein, thus definitely locating points on a vein. Combinations of outcrop points and bore-hole intersections may be used, just so three points on the same side of a vein are located. A vein has some thickness and is therefore bounded by two parallel planes, either one of which may be used to locate it. Since three points not in a straight line are sufficient to determine a plane, they definitely locate a vein. Any problem in relation to this

vein may be solved by treating the three points on the vein as a plane and by using the methods previously described for the solution of planes.

39.3. To Determine the Strike of a Vein of Ore, Having Given Three Points on the Vein.

ANALYSIS.

The three points, as given in the plan and in some elevation, determine a plane in those two views. A level line is drawn on this plane in the elevation view and projected to the plan, where its bearing may be read. The bearing of this level line is the strike of the vein.

Explanation (see Fig. 320).

The three given points which determine the vein are A, M, and N. The level line AC is drawn on the vein in the front elevation at a_2c_2 and projected to the plan at a_1c_1 where its bearing is found to be N 45° W. This is the strike of the vein.

40.3. To Determine the Dip of a Vein of Ore, Having Given Three Points on the Vein.

ANALYSIS.

The three points, as given in the plan and in some elevation, determine a plane in those two views. Since the dip angle is the same as the true slope angle of the plane, an auxiliary elevation showing the plane as an edge shows its dip. This view is drawn looking parallel to the strike.

Explanation (see Fig. 320).

A contour map of a small portion of ground is shown. The points M and N are points of outcrop on the upper surface of the vein. A vertical borehole intersects the upper surface of the vein at A and the lower surface at B. The three points, A, M, and N, establish the vein in both the plan and front elevation views. The strike is determined as in Section 39.3. The elevation view 3 shows the vein as an edge and also shows its true dip which is 34°. In this same view the point B on the lower surface is located also and the lower surface drawn parallel to the upper one. The distance between the two planes is the thickness of the vein and it may be measured in this view.

41.3. To Determine the Line of Outcrop of a Vein.

ANALYSIS.

The line of outcrop is the line of intersection of the plane of the vein with the irregular ground surface. Each point on this line may be found by intersecting a level line on the vein with a contour line on the ground at the same elevation. Several points must be found to determine the irregular line of outcrop.

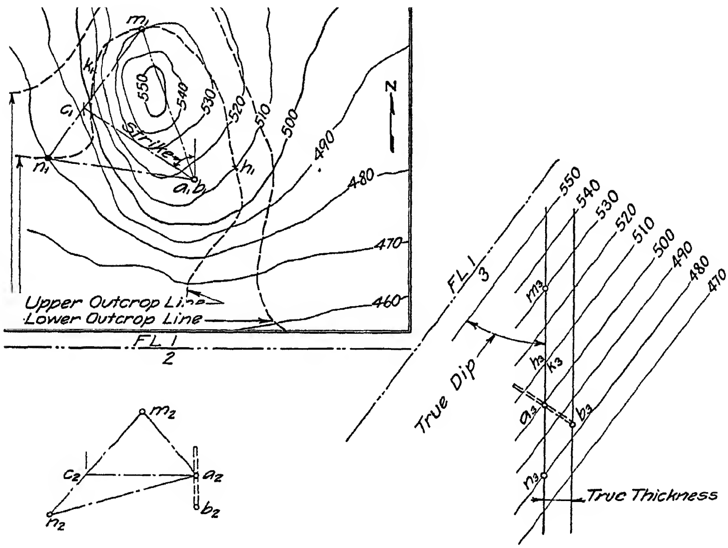


FIG. 320.—Strike, dip, and line of outcrop of a vein.

Explanation (see Fig. 320).

Referring to the upper plane AMN of the vein, the intersection of the 510-ft. contour with this plane as seen in view 3 gives the points h_3 and k_3 which are projected to the plan at h_1 and k_1. These two points are on the vein and also on the ground and are, therefore, two points on the line of outcrop which must be seen in the plan view. Other points determined in the same way at different elevations give the upper outcrop line. In the same manner the lower outcrop line is found. The extent of the vein is thus definitely shown within the limits of the map. Notice that the ore all lies between the arrowheads shown on the map. There is no ore outside of the lower outcrop line.

42.3. To Determine the Strike, Dip, and Thickness of a Vein by Two Nonparallel Boreholes.

ANALYSIS.

The two nonparallel boreholes would intersect both the upper and lower surfaces, giving two points on each surface of the vein. Connect the two points on the upper surface with a straight line. A view showing this line as a point shows the upper surface as an edge, although its direction is still unknown. Since the two

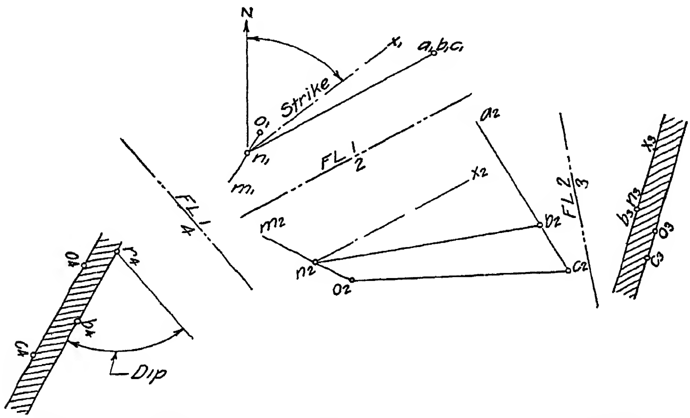


FIG. 321.—Strike, dip, and thickness by two nonparallel boreholes.

surfaces are parallel, the lower surface must also be an edge in this view and its position is determined by locating the two points on this surface. The upper surface may now be drawn parallel to the lower one and the thickness is apparent here. Either surface may be projected back to the plan to determine the strike and dip by the usual methods.

Explanation (see Fig. 321).

A vertical borehole at A intersects the upper surface at B and the lower surface at C. An inclined borehole at M intersects the upper surface at N and the lower surface at O. The line BN is therefore on the upper surface and it appears as a point in view 3. Points O and C on the lower surface are located in view 3 where they determine the edge view of the lower surface. The upper surface is drawn parallel to the lower surface and the true thickness is seen. To find the strike draw a level line NX on

the upper surface in view 2 and project it to view 3. This level line is now fixed in space and may be projected to n_1x_1 in the plan where the strike is measured. A new elevation, view 4, shows the dip. The true thickness may be seen again in this view if the lower surface is located here. If the two boreholes had been parallel, they would both have appeared as points in view 3 and the position of the surfaces would be indeterminate unless more information, such as the thickness of the vein and the direction of dip, were given.

43.3. Faults.

Veins of ore do not always continue indefinitely as plane surfaces, as has been assumed for the problems explained in the preceding sections. Sometimes they have what are known as "faults," which are caused by one part of the vein separating from the other part by either a sliding or a circular motion. These different kinds of faults introduce a large variety of problems of a more technical nature, that are without the scope of this book. Those who are more deeply interested in fault problems will find them treated in detail in mining texts such as the following:

HADDOCK, "Deep Bore-hole Surveys and Problems," McGraw-Hill Book Company, Inc.

LAHEE, "Field Geology," McGraw-Hill Book Company, Inc.

LINDGREN, "Mineral Deposits," McGraw-Hill Book Company, Inc.

PEELE, "Mining Engineers Handbook," John Wiley & Sons, Inc.

CHAPTER IV

REVOLUTION

1.4. Change-of-position Method.

Two entirely different methods are in use for solving drafting-room problems. All the problems that have been explained in the foregoing chapters of this text have been solved by the direct or change-of-position method. When using this method the draftsman imagines the object to be in a fixed position; he never thinks of it as being moved or turned around. If he wishes to obtain a different view, he must imagine himself occupying a different position in space in order to see what he wishes to see on the object. The object which is being drawn always remains stationary while the observer moves around it. This method gives an easier and a more direct solution for most practical problems, and it is used, even though unconsciously, by the large majority of draftsmen.

2.4. Revolution Method.

The alternate method for solving drafting-room problems is the revolution method, which requires the draftsman to remain in a fixed position and to imagine the object turned around so that he may obtain any desired view of it. Although this is not the most practical method for drafting-room use, some problems may be solved more easily by this method. For this reason the student should become thoroughly familiar with both methods, and thereafter should exercise his own judgment in selecting the one which is better suited to the solution of each particular problem.

Best results can be obtained by using a combination of the two methods, in order to avoid double revolution, a common source of error. The explanations which follow in this chapter are all based on this combination method.

3.4. Principles of Revolution.

Revolution is based on the following fundamental principles which must be thoroughly understood before any attempt is

made to solve problems. The use of these principles should insure a clear conception of what actually happens in space when revolution is performed.

I. *A point, when revolving in space, always revolves about some straight line as an axis.*

Caution. Do not try to revolve any point until you see clearly just how the axis lies about which the point is to be revolved.

II. *A point always revolves in a plane which is perpendicular to the axis, and its path in this plane is always a circle.* The radius of this circle is the shortest distance from the point to the axis.

III. *This circular path of the point will always appear as a circle in the view in which the axis appears as a point.*

In the front elevation view of Fig. 401, the axis AB appears as a point (a_2b_2), and the path of the point X in revolving about the axis AB appears as a circle. The true length of the radius is always seen in this view.

IV. *This circular path will always appear as a straight line at right angles to the axis, in the view which shows the true length of the axis.*

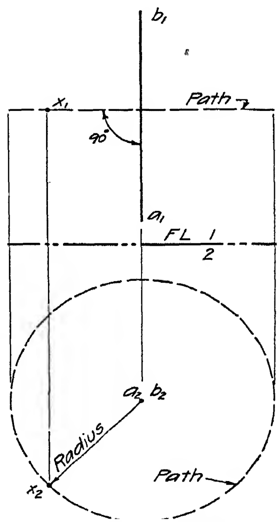


FIG. 401.—Illustrating the principles of revolution.

Since the plane of this circular path is perpendicular to the axis, it must show as an edge in looking at right angles to the axis to see its true length. The plan view of Fig. 401 illustrates this principle. The circular path of the point X is shown appearing as a straight line at right angles to the axis AB, which is in its true length in this view. In this case the plane of the path of motion is vertical, being at right angles to a level line.

4.4. Revolution Principles Illustrated.

The four principles that have been stated are fundamental, and they must be strictly adhered to whenever revolution is performed. When problems are to be solved in which the axis does

not appear as a point in either of the views given, a new view must be drawn which will show the axis as a point. The change-of-position method of drawing is employed in order to obtain the two views which will show the axis in its true length and as a point. These two views will then show the axis in such simple positions that the revolution may be easily performed in accordance with the four fundamental principles.

Figure 402 illustrates the statements made in the preceding paragraph. The point X and the line AB are given in the plan

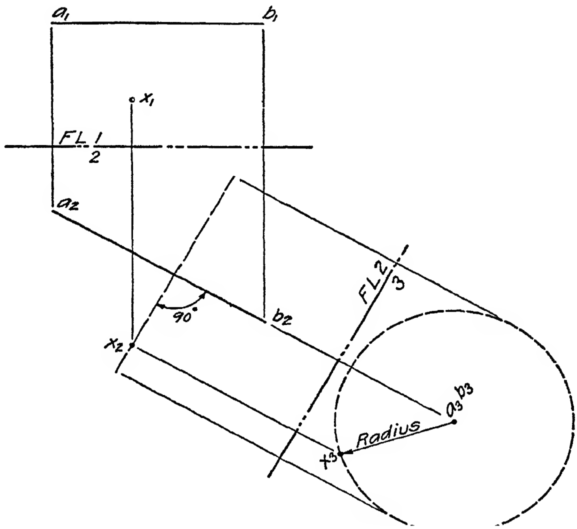


FIG. 402.—A point revolving about a frontal line.

and the front elevation and the point X is to be revolved about the axis AB. The axis appears in its true length (a_2b_2) in the front elevation, but it does not appear as a point in either of the views given. It is therefore necessary to draw a new view (view 3) which will show the axis as a point (a_3b_3). The revolution may now be performed by means of view 2 and view 3, in which the problem becomes exactly the same as the one shown in Fig. 401.

In some problems the axis does not even appear in its true length in either of the given views. This is the case with the axis AB, which is given in the plan and front elevations of Fig. 403. The point X is also given, and is to be revolved about

AB as an axis. It is then necessary to draw a new view (view 3) showing the axis in its true length (a_3b_3), and another view (view 4) showing the axis as a point (a_4b_4). The point X is also located in these two new views. This procedure again reduces the problem to its simplest form, that shown in Fig. 401, by performing the revolutions in views 3 and 4 only. If it is desired to stop the point X at some definite position as $x_4{}^R$ in view 4, the

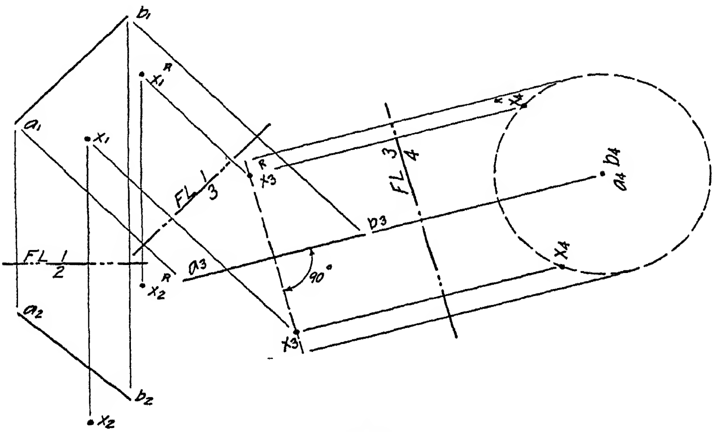


FIG. 403.—Point revolving about any oblique line.

new position in all views is easily determined by projection lines and measured distances from folding lines, as was explained in previous chapters.

5.4. Practice Problems.

See Chapter VIII, Group 28.

6.4. To Find the True Length of Any Line.

Revolve the line about an axis which intersects the line and which is parallel to the image plane that will show the true length of the line. If it is desired to revolve the line AB in Fig. 404 until its true length will be shown in the front elevation, the axis must be taken parallel to the front image plane, and usually it is taken in its simplest position, which is vertical. The revolution is then performed exactly as in Fig. 401, the four fundamental principles being adhered to. In this case the vertical axis is taken touching one end of the line AB, and the

other end of the line is revolved to the position b_1^R. The line AB now lies parallel to the front image plane; hence it appears in its true length in the front view.

The same process is followed if it is desired to show the true length of the line in any other view. In Fig. 405 the line AB is revolved so that its true length appears in the plan view, and the axis of revolution must be taken as a level line. This

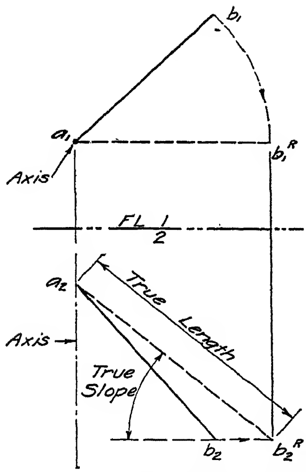


FIG. 404.—A line revolved about a vertical axis.

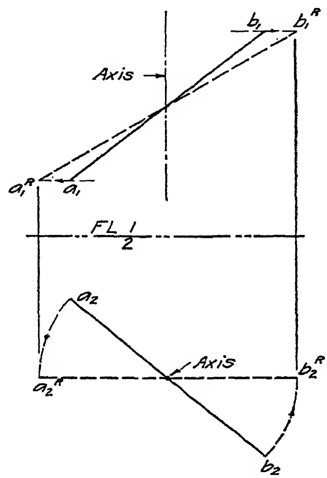


FIG. 405.—A line revolved about a level axis.

figure also illustrates the fact that the axis may intersect the line either at one end or at any point along the line; in the latter case both ends of the line are revolved. Strictly speaking, every point on the line is revolved that is not on the axis of revolution.

The true slope of a line will be seen in the elevation view that shows the true length of the line, provided the line has been revolved about a vertical axis, as in Fig. 404. Revolution of the line about any axis except a vertical axis will change its slope.

7.4. Practice Problems.

See Chapter VIII, Group 29.

8.4. To Find the True Size of Any Plane.

The plane must always be revolved about an axis that lies on the plane itself. This axis must also lie parallel to the image plane for the view which will show the true size of the plane. For example, if the plane is to be revolved so that it will show in its true size in the plan view, the axis must be taken as a line in the plane and parallel to the plan image plane; in this case, level. After this axis has been located, the procedure is that

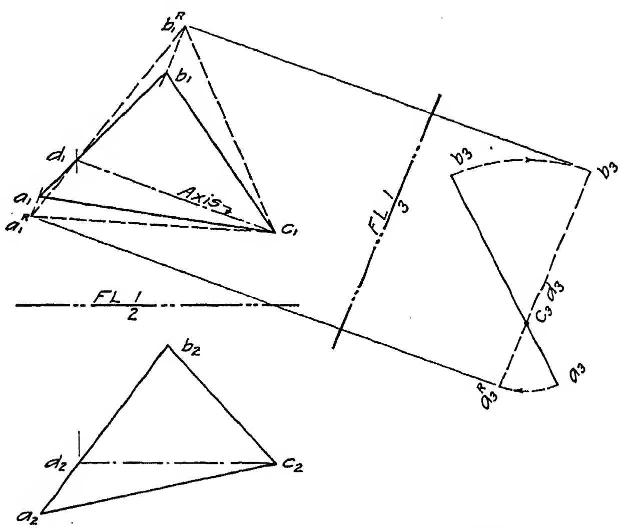


Fig. 406.—A plane revolved to a level position.

of previous problems, each point on the plane being revolved about the axis until the plane itself is level. In Fig. 406, the plane ABC is to be revolved to a level position. A level axis CD is chosen which lies in the plane. An extra view, 3, is drawn which shows the axis as a point (c_3d_3). By means of views 1 and 3, the solution is again reduced to the basic principles, as in Fig. 401. In view 3 the plane is shown revolved to a level position ($a_3{}^Rb_3{}^R$); while in this position, it shows in its true size ($a_1{}^Rb_1{}^Rc_1$) in the plan view.

In this same manner, any plane figure in any possible position in space may be revolved so as to show in its true size and shape in any desired view. Attention is called to the fact that a plane automatically appears as an edge in the view which shows

the axis as a point, since the axis is on the plane. This edge view must always be seen before the revolution can be performed.

9.4. Practice Problems.

See Chapter VIII, Group 30.

10.4. To Find the True Size of Any Dihedral Angle.

The dihedral angle between two planes is measured by a plane angle which is cut out of the two planes by a third plane passed perpendicular to the line of intersection of the two given planes.

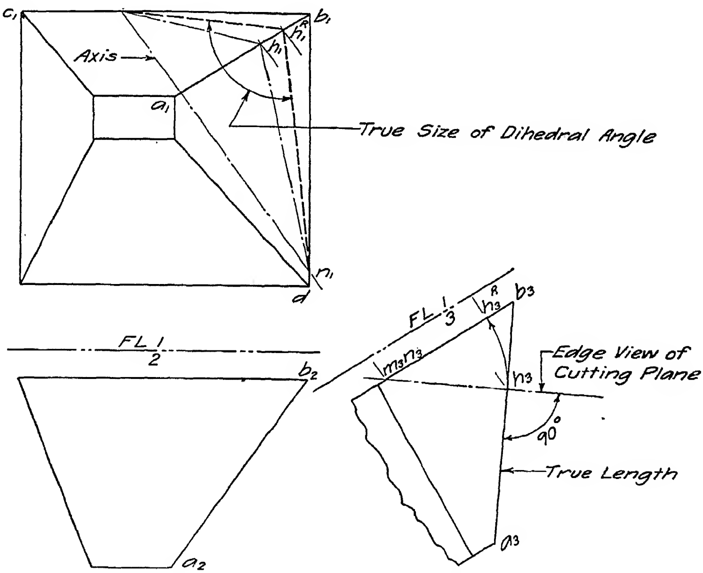


Fig. 407.—Dihedral angle.

The line of intersection of the two given planes is first determined. Then a plane is passed at right angles to this line of intersection, and the intersection of this third plane with each of the given planes is found. These two lines of intersection form the plane angle which measures the dihedral angle. The plane of this angle is revolved to such a position that its true size may be seen.

Figure 407 shows two given views of a hopper. It is desired to find the true size of the dihedral angle between the planes ABC

and *ABD*. The corner *AB* is the line of intersection of these two planes. A new view 3 is first drawn to show the line *AB* in its true length. A cutting plane is drawn perpendicular to the line *AB* at any point, such as *H*. This plane cuts the line *HN* out of the plane *ABD* and the line *HM* out of the plane *ABC*. The dihedral angle is measured by the plane angle between *HN* and *HM*. In order to see the true size of this plane angle, the cutting plane is revolved to a level position using any level line in this plane as an axis. The line *MN* was chosen as the axis and the vertex of the angle *H* moved from h_1 to h_1^R in the plan while the points *M* and *N* remained stationary. The true size of the dihedral angle is shown in the plan at $m_1h_1^Rn_1$.

11.4. Practice Problems.

See Chapter VIII, Group 31.

12.4. To Find the True Size of the Angle a Line Makes with a Plane.

The axis for performing this revolution should touch the line, and *it must be perpendicular to the plane*. If the line is revolved

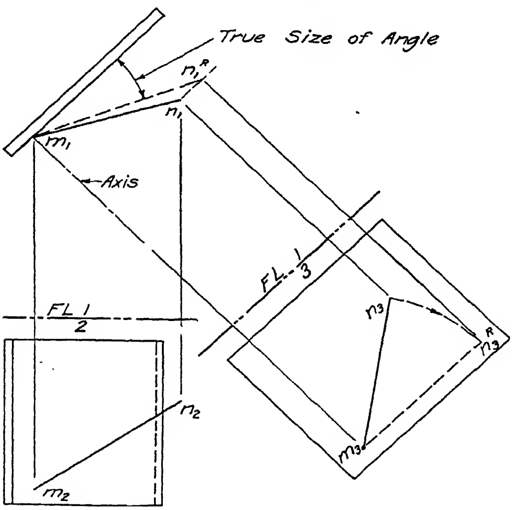


FIG. 408.—The angle a line makes with a plane.

about any other axis, the angle it makes with the plane does not remain the same. The line is revolved about the chosen axis

until it appears in its true length in the same view which shows the plane as an edge. Since the revolution is actually performed in the view which shows the axis as a point, this is also the view which shows the plane in its true size. In solving this problem for any possible position of the line and the plane in space, the following procedure must always be observed. First, draw a view of the line and the plane which shows the *true size of the plane* and the axis of revolution as a point. Then perform the revolution in this new view. The solution of this problem as shown in Fig. 408 should be self-explanatory. The plan view shows the edge view of the plane, the true length of the line found by revolving it about an axis perpendicular to the plane, and therefore the true size of the angle required. It should be noted that the distance from the point N to the plane did not change during the revolution. This condition must always hold if the angle with the plane is to remain the same in space.

13.4. Practice Problems.

See Chapter VIII, Group 32.

CHAPTER V

CONCURRENT NONCOPLANAR FORCES

1.5. Introduction.

The principles of orthographic drawing, which have been set forth so far in this text, furnish an interesting and practical method for the solution of the forces acting in a structure when the forces are concurrent but do not lie in the same plane. The method which is presented in this chapter is a graphical method for the solution of concurrent noncoplanar forces. It requires the use of a three-dimensional vector diagram which can be represented on paper, orthographically, only by two related views. The complete solution of a concurrent noncoplanar force problem requires the ability to perform the following operations.

1. To draw a view of a plane in which it appears as an edge.
2. To draw two related views of a closed line diagram having four or more sides, each with a fixed direction. This diagram is a three-dimensional figure.
3. To find the true length of each side and to measure it.

It is quite evident that the operations mentioned above involve only principles of drawing which have been previously explained and with which the reader should be thoroughly familiar.

It is also assumed that the reader is somewhat familiar with forces and vectors and with the solution of forces which lie in the same plane. Since the methods for the solution of coplanar and noncoplanar forces are exactly parallel, it is thought best to introduce here a very abbreviated review of coplanar forces and to give one example completely solved. The following is not intended to be a thorough discussion of coplanar forces, but is offered for the sole purpose of establishing a basis for the better understanding of noncoplanar forces. A more thorough explanation of coplanar forces may be found in any good text on Mechanics.

2.5. Definitions.

1. *Concurrent forces* are forces whose lines of action all pass through a common point.

2. *Coplanar forces* are forces whose lines of action all lie in the same plane.
3. *Noncoplanar forces* are forces whose lines of action do not all lie in the same plane.
4. A *resultant* of two or more forces is a force which may replace these forces and give the same effect.
5. An *equilibrant* of two or more forces is a force which will just balance these forces, or offset their effect, or put them into equilibrium. It is always equal in value to the resultant but opposite in direction.
6. A *vector* is a line which represents a vector quantity, such as a force. It has a definite length, direction and position.
7. A *force vector diagram* is a series of vectors, representing forces in a structure, laid down in consecutive order and direction, and each one starting where the preceding one stopped. This makes a connected figure, which must always be a closed figure if the forces are in equilibrium.

Since a force is a vector quantity, it may be represented by a vector with its direction parallel to the line of action of the force and with its length scaled to represent the value of the force. The vector diagram for coplanar forces cannot be drawn if more than two forces are unknown.

3.5. Solution of Coplanar Forces.

Figure 501 (*a*) shows a space drawing, to scale, of a boom carrying a load of 2,000 lb. on the cable *B* and held in position by the cable *A*. The pin *D*, through which three forces are acting in the same plane, is held in equilibrium by these three. Figure 501 (*b*) is an equilibrium, or free-body, sketch of the pin *D* with the three forces shown acting on it which hold it in equilibrium. This sketch is not drawn to scale and is used only to determine whether the unknown forces are tension or compression forces. Tension forces always act away from the joint and compression forces toward the joint in this sketch.

The force in the cable *B* is known, but the forces in the cable *A* and in the boom *C* are unknown and are to be determined. These three forces may be represented by vectors, which, when laid down one after another in any order, will form the closed vector diagram shown in Fig. 501 (*c*).

In order to facilitate the drawing of the vector diagram, a system of lettering known as Bow's notation is used. In this

system each space in the space drawing between two external forces (or loads) or between an external force and the frame is given a capital letter. Each space inside the frame or between two members is given a lower-case letter. An arrow is then drawn to indicate which way it is desired to read these forces around the pin *D*. It is immaterial which way the arrow reads, but, when it is once chosen, all forces should read around the pin in the same direction as the arrow. With the arrow point-

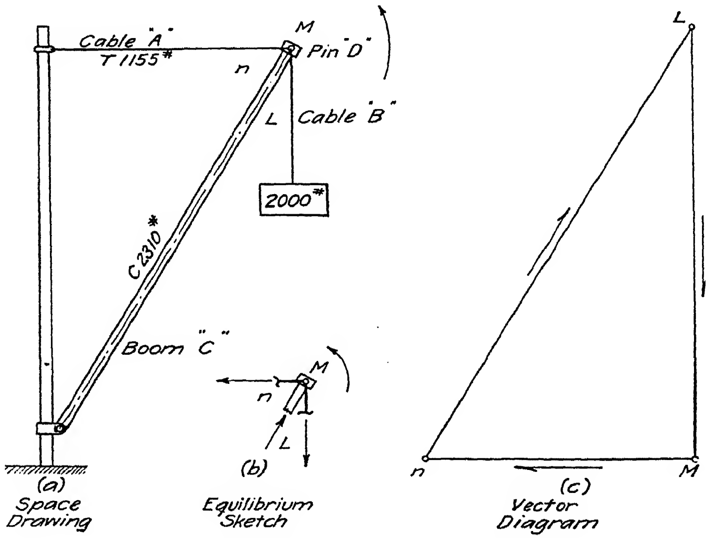


FIG. 501.—Solution of coplanar forces.

ing as shown in Fig. 501 (*a*), the vertical force reads *LM*, the level force reads *Mn* and the slanting force reads *nL*. If the arrow had pointed the other way the sequence of the notation would be reversed. The vertical force would read *ML*, the slanting force *Ln* and the level force *nM*.

The vector diagram may now be drawn, starting with the vertical force, because it is the only force whose value and direction are both known. This force is drawn as a vector in the vector diagram parallel to its line of action in the space drawing, and is measured equal to 2,000 lb. to some scale. This force acts down and, since its name is *LM*, it is labeled *L* at its beginning and *M* at its end. The next vector in order, following

the direction of the arrow, is Mn which is drawn from M parallel to the line of action of the level force in the space drawing. The length of this vector Mn is unknown. However, the next vector in order is nL which has a fixed direction and must return to L to close the diagram. Therefore the vector nL can be drawn through L, and point n is thereby determined. The vector diagram is now completed, and, since the entire diagram lies in one plane and shows in its true size, the two unknown vectors may be scaled to determine their values. Arrows are placed along each vector in the vector diagram, all arrows reading around the diagram in the same direction, which direction is determined by the arrow on the known vector LM. The arrows representing the direction of each of the unknown forces are next placed in the equilibrium sketch corresponding to their direction in the vector diagram. In Fig. 501 (*b*) the force Mn is seen to be a tension force and the force nL is seen to be a compression force. The value of each force, together with a capital T or C to indicate tension or compression, is usually recorded along the proper member in the space drawing as shown.

4.5. Noncoplanar Forces.

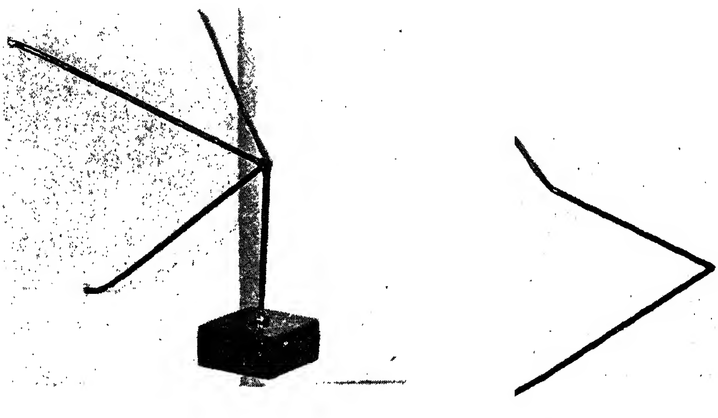
The identical methods explained in the preceding section for solving coplanar forces may also be used for solving noncoplanar forces. However, it must be kept clearly in mind that, with noncoplanar forces, the third dimension enters into the problem. A line will now have to be drawn in two views to fix its position in space, and it may not show in its true length in either of these views. Two views will now be necessary to represent the structure in the space drawing, and two views will also be necessary to completely represent the vector diagram. It is therefore most essential that the reader grasp the idea that there are two separate and distinct objects side by side in space, namely, the structure itself and its corresponding vector diagram, which is a closed figure having each line parallel in space to the respective member of the structure in which it acts. Figure 502 shows a photograph of a small model of a hanging frame and its corresponding vector diagram. Section 6.5 gives a complete explanation of the solution for this same frame.

Briefly stated, the solution consists of drawing a plan view and some elevation view of the structure itself and its vector diagram,

as they stand side by side, and finding the true length of each line of the closed vector diagram. Before a detailed solution is presented, several statements will be given for the express purpose of checking up the ability of the reader to think clearly and properly in three dimensions, and of directing this ability toward the solution of three-dimensional vector diagrams.

5.5. Principles.

The following basic principles regarding the relations existing between orthographic views and vector diagrams are stated below



(a) (b)
FIG. 502.—(a) Structure with a load. (b) Vector diagram.

without proof. They should be carefully studied and thoroughly understood before the solution of a problem is attempted.

1. The vector diagram for any system of forces in equilibrium must be a closed figure.
2. A three-dimensional vector diagram will appear as a polygon in all orthographic views.
3. Each line, or vector, of a vector diagram in space lies parallel to the corresponding member in space.
4. If two lines are parallel in space they will appear parallel in all orthographic views.
5. In the same view of any space drawing and its corresponding vector diagram, each line of the vector diagram will be parallel to its corresponding member in the space drawing.

6. The true force in any member is shown in the vector diagram only when the member itself appears in its true length in the space drawing.
7. The true length of each line of the vector diagram must be seen before it can be scaled.
8. All vertical vectors show in their true lengths in all elevation views.
9. All horizontal vectors show in their true lengths in the plan view.
10. The true lengths of all other vectors may be found by orthographic methods.
11. Noncoplanar problems usually have one or more known forces and three unknown forces. Four unknown forces are impossible of solution. Two unknown forces with only one known force give an impossible condition for equilibrium unless the three forces lie in the same plane. There must be not less than four forces if they are noncoplanar and in equilibrium.
12. With three unknown forces, the easiest solution is made possible when two of these unknown forces appear in the same line or when one unknown force appears as a point in some view.
13. The two unknown forces which appear in the same line in some view *must be kept consecutive in the vector diagram*, and should be the last two forces to be drawn in the vector diagram.
14. If one unknown force appears as a point, it may occur anywhere in the sequence of forces in the vector diagram.
15. Bow's notation should be used *in one view only*, *never in two views*.

6.5. Solution of Special Case.

Figure 503 shows the plan and front elevation views of a simple hanging frame carrying a 1,000-lb. load suspended at the point A. This frame is shown in the photograph in Fig. 502. It is desired to solve the three unknown forces acting in the members AB, AC, and AD. This is a special case, because two of the members, AB and AD, appear in the same line in one of the given views.

The plan and front elevation views of the structure itself are first drawn to a definite scale, as in Fig. 503 (a). This is the

space drawing. The spaces between the members are lettered using Bow's notation, which is placed on the plan view only. The vertical load appears as a point in this view and it is shown bent to one side just for convenience in placing notations around it. The true direction of the line of action of the force in this member is not altered by this temporary bending. In this case, if it were bent to the right it would lie between the two forces in *AB* and *AD* which must be kept consecutive and the solution

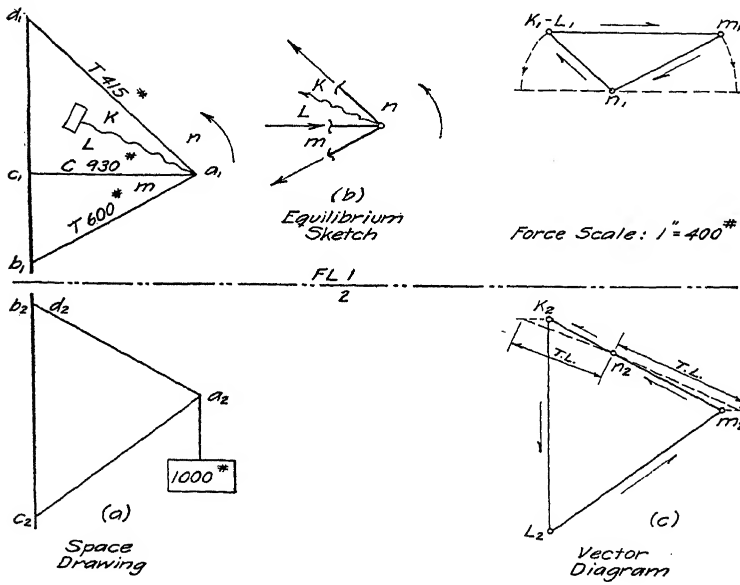


Fig. 503.—Solution of noncoplanar forces.

would be much harder. Therefore the vertical load is bent as shown. (See Section 5.5, principle 13.) The curved arrow is placed in the plan view to give the order of reading the forces around the joint *A*.

The vector diagram is then drawn to some assumed scale, starting with the only known vector *KL* and drawing it in both the plan and elevation views, as in Fig. 503 (*c*). Since the vector *KL* is vertical, it will appear as a point in the plan and in its true length in the front elevation. Each vector should be lettered at both ends and in both views as soon as it is drawn. The next vector in order, *Lm*, is drawn from *L* in both views of the vector diagram. Its direction in each view of the vector diagram must

be parallel to the direction of the member *AC* in the corresponding view of the space drawing. But the length of the vector *Lm* is unknown and therefore the position of *m* is still undetermined.

The remaining two vectors in order are *mn* and *nK*. These two forces appear in the same line in the front elevation of the space drawing, and their vectors must therefore have the same direction in the front elevation of the vector diagram. Since they are consecutive, they will actually be on the same line in the front elevation of the vector diagram and may be drawn in that view as one line, to close the diagram back to the starting point *K*. This determines point *m* in both views but still leaves point *n* undetermined. Since the point *m* is now known in the plan view, the remaining two vectors, *mn* and *nK*, may be drawn in this view to close the diagram back to the starting point *K*. This can be done because the direction of both vectors in the plan view is known. The intersection of these two vectors determines point *n*, which may also be projected to the front elevation and located there. The four-sided vector diagram is now closed and complete in both views, and it is therefore a definitely fixed figure in space as shown in the photograph in Fig. 502. It remains only to find the true length of each line or vector in the diagram and to scale that length to determine the value of each force. Arrows should be placed to indicate the direction of action of each force in each view of the vector diagram, as in Fig. 503 (*c*).

An equilibrium sketch, showing the forces acting on the joint at *A*, should be drawn as shown in Fig. 503 (*b*). This sketch is just a reproduction of a portion of the view of the space drawing in which the notation has been placed. The plan view was used in this case. The actual member is shown broken off and the line of action of the force acting in that member is also shown. Arrows, to indicate the direction of action, are placed on each force in the equilibrium sketch exactly as they point in the corresponding view of the vector diagram. The forces *nK* and *mn*, acting away from the member in the equilibrium sketch, are therefore tension forces and the force *Lm* is a compression force. The value of each force, with *T* or *C* to indicate tension or compression, should be placed along the member in which it acts in one view of the space drawing. The plan view was chosen for recording these values in this problem, because it would show them more clearly.

Note: If Bow's notation is used in this way, any force which is designated by one or more lower-case letters acts in a member of

the structure, and any force which is designated by two capital letters is an external force, or a load.

7.5. Solution of General Case.

Attention is called to the fact that in the problem which has just been solved in Section 6.5 the frame was placed in such a position that two of its members appeared in the same line in the front elevation. This may be considered to be a special case, for not every structure might happen to have two of its members appearing as a line in either of the given views. However, by the method of Section 12.2 for finding the edge view of any plane, it is very easy to draw a new elevation view of any two intersecting lines, in which they will appear as the same line.

In order to solve a general case, a new elevation view of the structure must first be drawn in which two of the members appear in the same line. The plan and this new elevation view are treated exactly the same as though they were the given views, and two views of the vector diagram are drawn corresponding to these two views. From this point the solution is exactly the same as for the special case in Section 6.5, and for that reason the drawing for this solution in detail is not given.

Any possible general case having three unknowns may be reduced to the special case and solved as such by simply drawing some new view showing two of the unknowns in the same line. If there should happen to be more than one known load, as in airplane work, these known forces should always be kept consecutive and should be drawn first in the vector diagram. In some cases it might be easier to use two elevation views or possibly an elevation and an inclined view instead of the plan and elevation views. Any two related orthographic views will enable the problem to be solved by this method, provided the vector diagram is represented in two views corresponding to those chosen for the space drawing.

8.5. Solution by Seeing One Unknown as a Point.

The plan and front elevation of a frame are given as in Fig. 504. The frame has a horizontal load and the member AB is level, showing in its true length in the plan. A new elevation, view 3, is drawn of the entire frame so as to show the member AB as a point. Bow's notation is placed in the plan. Two views of the vector diagram are constructed to correspond with views 1 and 3

of the space drawing. The known force MN is drawn first and then the force Nh, which appears as a point in the elevation. The two closing forces can then be drawn in the elevation, fixing the point K, which is projected to the plan. The plan can then be completed and the vector diagram closed. The problem is completed from here just the same as the one in Fig. 503.

The solution could also have been made with a view showing AC (or AD) as a point, but an extra view showing its true length

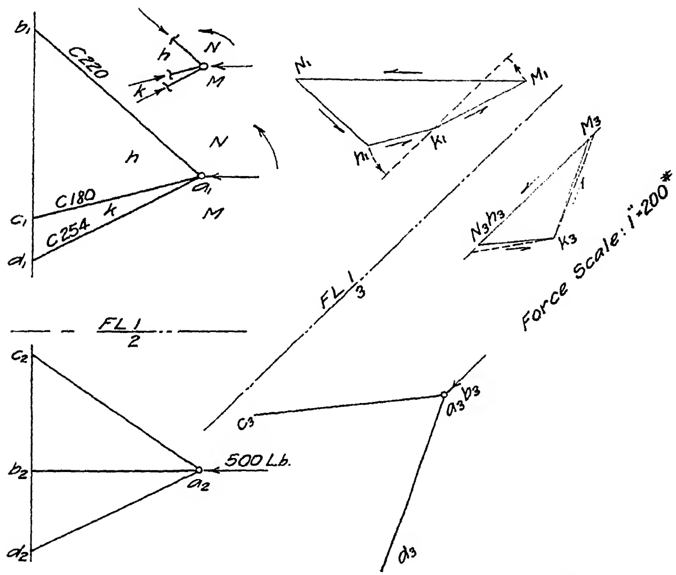


Fig. 504.—Solution by seeing one unknown as a point.

would be necessary before it could be seen as a point. Generally speaking, this method should be used if one of the members shows in its true length in one of the given views; otherwise, the method of Section 7.5 should be used.

9.5. Practice Problems.

See Chapter VIII, Group 33.

10.5. Alternate Method.

Principle 12 in Section 5.5 stated that the easiest solution for noncoplanar force problems is made possible when two of the unknown forces appear in the same line or when one of the

unknowns appears as a point in some view. While this condition is highly desirable in order to furnish the easiest solution, it is not absolutely necessary, and any general case may be solved just as it stands without obtaining the new view of the space drawing. This method depends directly upon the principle explained in Section 12.3, second method.

In Fig. 505 are given two views of a tripod structure as shown in the space drawing. After insertion of the notation and the

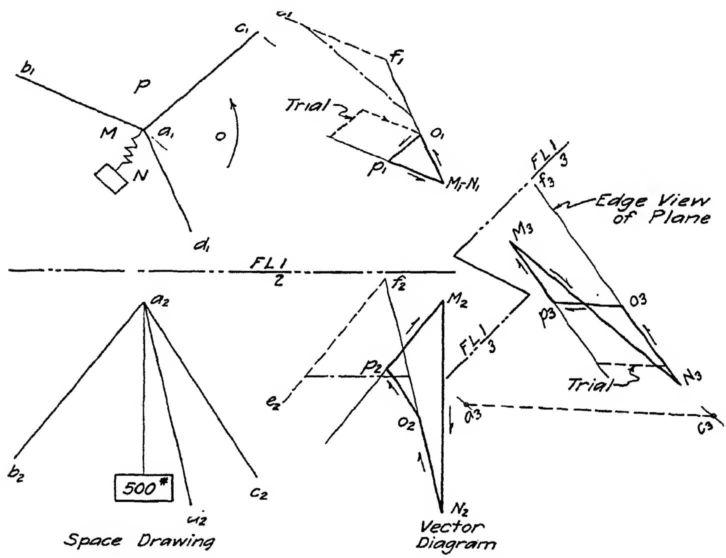


FIG. 505.—Solution of noncoplanar forces. Alternate method.

arrow, the two corresponding views of the vector diagram are drawn as far as possible, starting with the known vector MN. The next vector in order, No, can be drawn from N in both views. Also the vector preceding MN, or pM, can be drawn from M in both views. But the remaining vector op, between No and pM, cannot be drawn in either view. Its direction is known, but its position is unknown in both views.

A plane, NFE, is drawn containing the vector No and parallel to the vector pM in the vector diagram, the line EF being taken parallel to pM so that it intersects No at F. A new elevation, view 3, of the vector diagram is drawn which shows the plane NFE as an edge and the vector pM appearing parallel to this

plane, as it was assumed. The member AC is projected from the space drawing into this new elevation view and is shown at a_3c_3. The vector op must be parallel to the member in which it acts, and it is therefore parallel to a_3c_3 in this view. The position of op is still unknown, but its direction is known in this view. Therefore, regardless of its position, its length is definitely fixed, since No and pM appear parallel. Accordingly, the vector op is drawn in, at any trial position, in view 3 as shown. This fixes its length. It is then projected to the plan view of the vector diagram with one end on the line of the vector pM, and the other end is determined by projection. This is just the trial position, but its length and direction in the plan are both definitely known now. This length is then moved over from the trial position of op, keeping it parallel to the trial position and with one end on pM, until the other falls on Nf. This procedure fixes the position of the vector op in space and makes the vector diagram a closed figure. The other views of op may be determined by projection, and when it has been located in all views its direction must check with its known direction, which is parallel to the member in which the force acts. In Fig. 505 the solution has not been completed beyond this point. The true lengths of each vector are still to be found and scaled, the equilibrium sketch is to be made, and the problem is to be completed exactly like the one preceding.

The method just given has a slight advantage in that it takes less room on a drawing sheet and gives a better placing of views on the sheet, but it is not so easy to see.

11.5. Practice Problems.

See Chapter VIII, Group 33 (by the alternate method).

12.5. Resultant and Equilibrant.

The three-dimensional vector diagram may also be used to determine the value and direction of the resultant or the equilibrant of three or more concurrent, noncoplanar forces. If three or more forces which are not in equilibrium are known in value and direction, the vector diagram may be constructed in both views, but it will not be a closed figure. It could be closed with one line, which would be fixed in both views and therefore fixed in space as to its value and direction. The value of this closing vector is the same for both the resultant and the equilibrant. If its direction is taken around the vector diagram, reading the

same as all the other arrows, it will be the equilibrant or the force which will just hold the other forces in equilibrium. If its direction is taken to read opposite to all the other arrows in the vector diagram, it will be the resultant of the given forces. In other words, all the given forces could be replaced by this one resultant force, whose value and direction are now known, and the effect would be the same.

Another method for finding the resultant of three concurrent noncoplanar forces is to treat any two of them as a plane in which their resultant must lie. On the true length of each of these two members in the space drawing the value of the force acting in that member is laid out to some scale, and these lengths obtained are shown in both views. The parallelogram of these two forces is completed to determine their resultant, in both views. This resultant may now be considered as replacing the first two forces. It and the third force are next treated as another plane, and their resultant is found in exactly the same manner. This last resultant obtained is the resultant of all three given forces. Its value is also that of the equilibrant, whose direction is just the reverse.

13.5. Practice Problems.

See Chapter VIII, Group 34.

14.5. Other Applications.

The methods for solving three-dimensional vector diagrams which have been set forth in this chapter have all been illustrated by problems dealing with forces acting in frame structures. These methods might find their largest use in problems of this nature. However, in many other practical situations in engineering three-dimensional vector diagrams may be used. In electrical engineering this method applies to the solution of electromagnetic or electrostatic vectors. Combinations of velocities in mechanisms or in space travel may be treated in this way. Likewise moments, rates, and any other quantities which may be represented by vectors not lying in the same plane, may be determined; or if they are known their resultant may be determined by these methods. This graphical solution is much faster, in most cases, than the more complicated mathematical solution, and with proper care it will give results which are sufficiently accurate.

CHAPTER VI

CURVED LINES AND SURFACES

1.6. Introduction.

The majority of the objects that an engineering draftsman is called upon to draw are bounded by straight-line corners and plane surfaces. All of the problems which might arise in working with these can be handled by the methods that have been explained in the previous chapters.

However, many situations confronting a draftsman require a knowledge of curved lines and curved surfaces for the correct representation of an object or for the proper solution of a problem. Many varieties of cams, gears, and screw conveyors depend entirely on curved lines for their very operation. Modern concrete construction often includes a warped surface on a dam, an irrigation canal, or a syphon. Many varieties of hydraulic and irrigation pipe problems include combination surfaces made up of planes, cones, and cylinders.

An engineer should be able to tell, at a glance, whether or not it is possible to develop a certain surface. He should also know how to develop any surface which can be developed. In order to have a thorough foundation in the fundamental subject of drawing, he must at least be familiar with curved and warped surfaces and with the solution of problems relating to them. The material in this chapter is not presented with any intention of covering this subject completely. Sufficient explanations are furnished to give a reading knowledge of the various types of surfaces and to enable one to solve the ordinary problems with which he might be confronted. The major emphasis will be placed upon problems dealing with cylinders and cones, because these occur more frequently in engineering practice.

In general, the change-of-position method is still the most natural one to use. However, in some cases, such as finding the true length of a series of lines, the revolution method will be found to be very much easier and faster. Judgment should be exercised as to which method is the more efficient for each solution attempted.

2.6. Curved Lines.

Curved lines fall naturally into two general classes, which are determined entirely by the third dimension. These two classes are *lines of single curvature* and *lines of double curvature.*

3.6. Single-curved Lines.

Lines of single curvature lie wholly in one plane and are therefore just two-dimensional or plane curves. There may be an infinite number of these curves, determined by an infinite number of mathematical equations. Some of the most common are the circle, ellipse, parabola, and hyperbola. These four curves are called conic sections. Then there are the trigonometric curves, the most common of which are those of the sine, cosine and tangent. Involute and cycloidal gear teeth and irregularly shaped cams furnish other practical illustrations of single-curved lines.

The detailed explanations for the actual construction work involved in drawing single-curved lines are omitted here, because they are usually taught in elementary drawing courses and because they may be found in almost every current text on engineering drawing.

4.6. Double-curved Lines.

Lines of double curvature do not lie wholly in the same plane. They are three-dimensional or space curves. There may also be an infinite number of these curves, which are determined by an infinite number of mathematical equations. Most of these curves do not have a practical physical application but occur more often in pure mathematical calculations. The only curve of this class which will be discussed further will be the helix.

5.6. Helix.

The helix is a vital part of practically every machine in existence. The statement may be safely made that there is no automobile, airplane, ship, train, engine, or power-driven machine that does not have a helix somewhere in its structure, in the form of either a screw-thread or a coil spring.

The helix is generated by a point moving around an axis and parallel to that axis at the same time. The point usually (though not necessarily) moves at a uniform rate in both directions. If it remains a fixed distance away from the axis, a *cylindrical helix* is generated, since the point will be traveling around a cylinder.

If the point moves on the surface of a cone so it goes around the axis as it approaches the vertex of the cone, it generates a *conical helix*. A conical spring is an example.

Figure 601 shows the plan and elevation of a right-hand cylindrical helix which is generated by a point starting from A and traveling once around the cylinder whose diameter is D while it is traveling a distance L parallel to the axis of the cylinder. The distance L is called the *lead* of the helix. The *pitch* of a helix

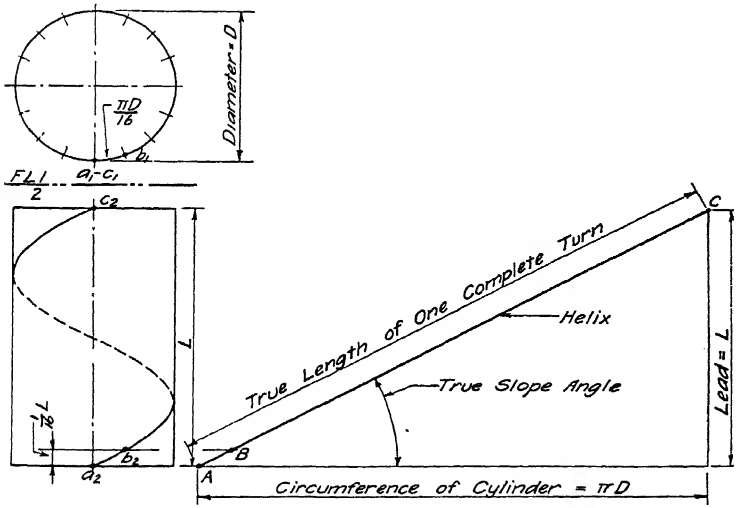


FIG. 601.—Right-hand cylindrical helix.

on a double, triple, or quadruple screw thread is the distance from a point on one thread to the corresponding point on the adjacent thread.

The curve is drawn by dividing the distance around the cylinder into any number of equal parts and by dividing the lead into the same number of equal parts (16 in this case). As the point A travels one-sixteenth of the distance around the cylinder it travels one-sixteenth of the lead, gaining the position at B. By taking each position separately the construction is easily made, and the entire curve may be drawn as shown.

If a triangular paper is cut out, like the one shown in Fig. 601, having a base equal to the circumference of the cylinder and an altitude equal to the lead of the helix, the hypotenuse

of the triangle is the actual or true length of one complete turn of the helix. The angle shown is the true slope angle of the helix, and the tangent of this angle is seen to be $L/\pi D$. When the triangular paper is wrapped around the cylinder the hypotenuse coincides with the helix, because the helix has a uniform slope and must in consequence be a straight line when it is developed. This triangle gives a clearer idea of the helix than can be obtained by any other means.

6.6. Practice Problems.

See Chapter VIII, Group 35.

7.6. Curved Surfaces. Definitions.

1. A *surface* is the path of a moving line (except when the line is straight and moves endwise like an arrow).
2. The *generatrix* is the line which moves to generate a surface.
3. An *element* is the generatrix at any one position. A curved surface may therefore be considered as being made up of an infinite number of elements, or different positions of the generatrix. (An element is also considered to be any straight or curved line on the surface.)
4. A *directrix* is a line or a plane which guides or defines the motion of the generatrix. There may be one, two or three directrices for the same surface.
5. A *ruled surface* is one which may be generated by a straight line. A straight-edged ruler may be laid on a ruled surface so that it will touch the surface for its entire length.
6. A *double-ruled surface* is a surface through any point of which it is possible to have two intersecting straight-line elements.
7. A *single-curved surface* is a ruled surface that can be developed or rolled out into a plane.
8. A *warped surface* is a ruled surface that can not be developed.
9. A *double-curved surface* is a surface which can be generated only by a curved line and which has no straight-line elements.
10. A *surface of revolution* is the path of any line which revolves about a straight line as an axis. The line that revolves may be straight or curved. A straight line revolving in this manner will generate a single curved surface of revolution if it is parallel to or intersects the axis, giving a cylinder of revolution or a cone of revolution respectively. But if the straight line is neither parallel to nor intersects the axis,

TABLE II.—OUTLINE OF CURVED SURFACES

		Name	Generatrix		Developable or not
			Kind of line	Kind of motion	
Ruled	Single-curved	Cylinder	Straight line	Touches single-curved line; remains parallel to straight-line directrix	Yes
		Cylinder of revolution	Straight line	Revolves about straight-line directrix to which it is parallel	Yes
		Cone	Straight line	Touches single-curved line; intersects straight-line directrix	Yes
		Cone of revolution	Straight line	Revolves about straight-line directrix which it intersects	Yes
		Convolute	Straight line	Remains tangent to any double-curved line	Yes
		Helical convolute	Straight line	Remains tangent to a helix	Yes
	Warped	Helicoid	Straight line	Touches helix and its axis; makes constant angle with the axis	No
		Hyperbolic paraboloid	Straight line	Touches two nonintersecting nonparallel straight lines; remains parallel to a plane	No
		Conoid	Straight line	Touches one straight line and one curved line; remains parallel to a plane.	No
		Cylindroid	Straight line	Touches two curved lines; remains parallel to a plane	No
		Hyperboloid of revolution of one sheet	Straight line	Revolves about an axis which is nonintersecting and nonparallel	No
Double-curved	Surfaces of revolution	Sphere	Circle	Revolves about its diameter	No
		Torus or Annulus	Circle	Revolves about any straight line except its diameter	No
		Ellipsoid (prolate)	Ellipse	Revolves about its major axis	No
		Ellipsoid (oblate)	Ellipse	Revolves about its minor axis	No
		Paraboloid	Parabola	Revolves about a symmetrical axis through its focus	No
		Hyperboloid of two sheets	Hyperbola	Revolves about an axis through both foci	No
	Irregular	Unnamed	Any curved line	Moves along any other curved line	No

the surface generated will be a warped surface called the hyperboloid of revolution of one sheet (or one piece).

11. A *right section* of a surface of revolution is a plane section which is perpendicular to the axis; it must always be a circle.

Table II gives a classification of most of the surfaces commonly occurring in engineering work, and it will show quickly the general class to which a surface belongs, the way in which it is generated, and whether or not it can be developed.

8.6. Cylinder.

A cylinder is generated by a straight line moving around another straight line and always remaining parallel to it. It is usually considered to be a closed surface, which means that the generating line returns to the place from which it started. The

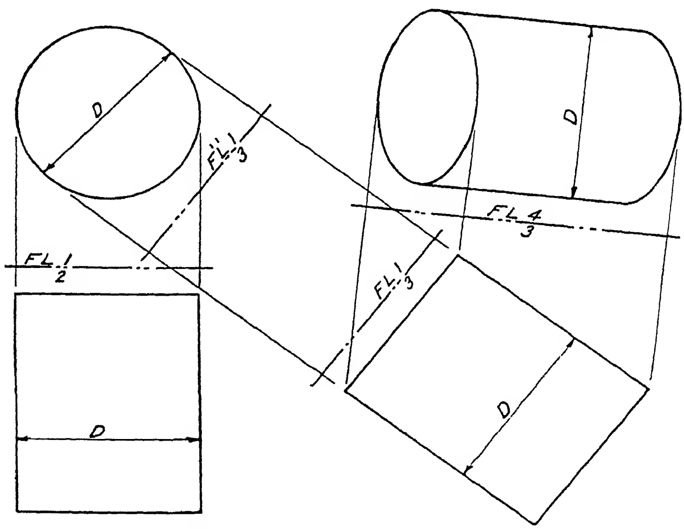


FIG. 602.—Cylinder of revolution.

path of any point on this generating line may be any curved line and does not have to be a circle. If this path should be a circle in a plane perpendicular to the axis of the cylinder, the surface would be a *cylinder of revolution*, or just a round cylinder as in Fig. 602.

Most cylinders in practice are round, as in the case of tanks, pipes, etc. For this reason the common conception of cylinders

is that they have to be cylinders of revolution. This is a false conception. The cylinder is a cylinder of revolution only when the right section is a circle. If an oblique section of a cylinder is a circle, the shape of the right section of the cylinder is an ellipse, as in Fig. 603. This cylinder is an elliptical cylinder.

9.6. Representation of a Cylinder.

Although a cylinder is considered to have an infinite number of elements, only two elements are drawn in any view and these

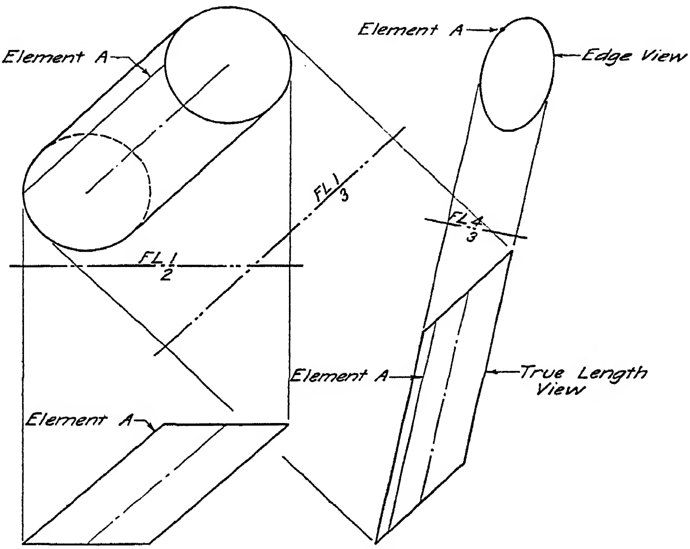


FIG. 603.—Oblique elliptical cylinder.

are the two that project in the extreme outside position on each side. They are called the extreme elements. The cylinder is represented in any view by drawing its upper or lower base, or both bases, and the two extreme elements in that view. This fact is shown in Fig. 603. The extreme element A in the front view is not an extreme element in any other view.

In order definitely to locate any element on a cylinder for purposes of projection, some point on the element is always used which lies in one of the given bases or in any other plane section of the cylinder. This one point may be projected to different views, and the element may then be drawn. It is necessary to

locate only one point on each element in any view, because all the elements project parallel to the axis in every view.

Since all the elements on the surface of a cylinder are parallel to the axis, the following theorem may be stated.

Theorem **10.** *The view of a cylinder which is drawn showing the axis as a point will show the cylinder surface as an edge and will give the true shape of its right section* (see view 1, Fig. 602, and view 4, Fig. 603).

Also it may be proved that any oblique view of a cylinder of revolution will show it always to have the same width. This fact is a very useful one and is restated as a theorem.

Theorem **11.** *A cylinder of revolution will have a width equal to its diameter in every possible view* (see Fig. 602).

10.6. Practice Problems.

See Chapter VIII, Group 36.

11.6. To Find Where a Line Pierces a Cylinder.

First Method: By Auxiliary Views.

ANALYSIS.

A view of the cylinder is drawn looking parallel to its axis and showing the cylinder as an edge (by Theorem 10). The given line is then located in this same view. The two piercing points where the line passes through the cylinder will be apparent in this edge view and may be projected back to all other views. The solution by this method is not shown, but it would be complete in view 4 of Fig. 603 if the given line was located in that view.

Second Method: Using Only the Two Given Views.

ANALYSIS.

It is usually easier to solve problems in curved surfaces by using straight-line elements on the surface. They should be used whenever it is possible. In order to cut straight-line elements on the cylinder, a cutting plane will have to be passed parallel to the axis and to all the straight-line elements. This cutting plane will have to contain the given line also, and it will contain two straight-line elements where it cuts the surface of the cylinder. These two elements lie in this same plane with

the given line, which will have to either intersect them or be parallel to them. If it happens to be parallel to them it is parallel to the cylinder and cannot pierce it. If it intersects them, the two points of intersection are the required piercing points. The given views will always allow one to determine

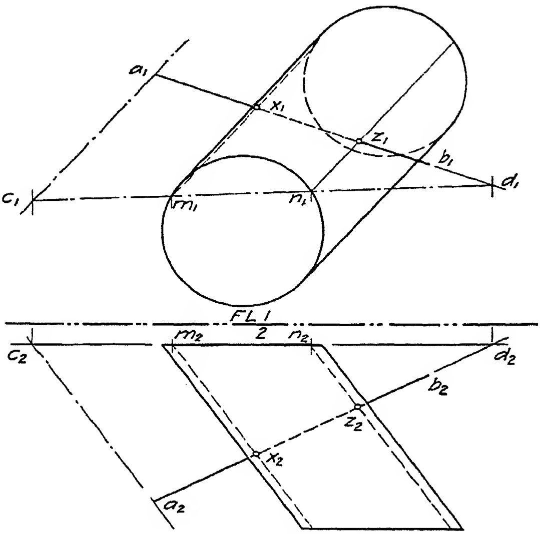


FIG. 604.—Line piercing a cylinder.

by inspection whether or not the line is parallel to the cylinder (by Theorem 1, Section 2.3).

Explanation (see Fig. 604).

The oblique cylinder and the line AB are given in the two views, and the points at which the line AB pierces the cylinder are required.

In order to use straight-line elements on the cylinder, a plane is passed containing the line AB and parallel to the axis of the cylinder. The line AC is the auxiliary line used, and the cutting plane BAC is the desired plane. This cutting plane intersects the plane of the upper base of the cylinder in the line CD and cuts across this base of the cylinder at the points M and N. These two points are the upper ends of the two straight-line elements, which also lie in the plane BAC. The two elements are drawn in both views, since they have to be parallel to the axis;

the points where they intersect the line AB are the required piercing points X and Z. These two points, which have been determined independently in both views, should also check by projection between views. The plane of the lower base could also have been used for determining the two straight-line elements.

12.6. Practice Problems.

See Chapter VIII, Group 37.

13.6. Plane Section of Cylinder with Vertical Axis.

First Method: By Auxiliary Views.

ANALYSIS.

A new elevation view is drawn showing the given plane as an edge and showing the cylinder as well. This new view will

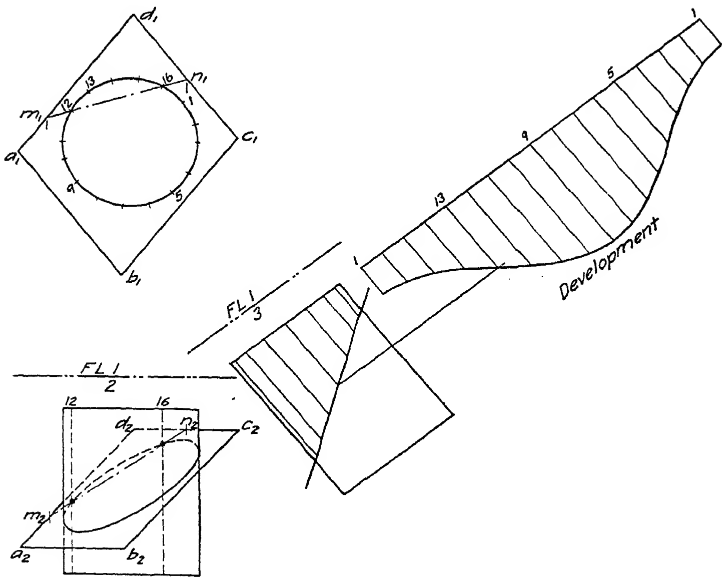


FIG. 605.—Plane section and development of a cylinder.

indicate the points at which each element of the cylinder pierces the plane or is cut off by the plane. This piercing point on each element can then be located on the proper element in any

other view by projection. When all the piercing points have been located in this manner in any view, the entire curve may be drawn.

Explanation (see Fig. 605).

The cylinder and the plane *ABCD* are given in plan and front elevation. The new elevation is drawn looking parallel to the level line *AB*, in order to show the plane as an edge. The cylinder is also shown in this view. The cylinder is divided into sixteen elements equally spaced around the circumference, the division being made in the edge view of the cylinder. It is usually safer to number these elements (in pencil) while the problem is being solved. These 16 elements are located in the new view, and it is apparent where each element is cut off by the plane. These piercing points are all located in the front elevation. A line connecting them gives the plane section across the cylinder; in this view it is a curved line, as shown. Any point upon it will be visible if it lies on a visible element of the cylinder. Exactly half of the elements will be visible in any view; they will be the ones that are closest to the observer.

Second Method: Using Two Views Only.

ANALYSIS.

By the method of finding where a line pierces an oblique plane the point may be determined at which each element on the cylinder pierces the plane. A line connecting these points will give the desired curve that the plane cuts on the cylinder.

By the method of Section 18.3, vertical projection planes are drawn containing one or two elements at a time. One of these planes is shown at *MN*, taken so as to contain the two elements numbered 12 and 16. The points at which these two elements pierced the plane are found in the front view and marked with two small circles. The piercing points of all 16 elements are found in the same way; they determine the required curve.

Both of the methods just explained are applicable to all cylinders, whether they are cylinders of revolution or not.

14.6. Practice Problems.

See Chapter VIII, Group 38.

15.6. Development of Surfaces.

To develop a surface means to lay it out into a plane like a flat piece of metal. All containers that are to be made out of sheet or plate metal must first be laid out on the metal when it is in flat sheets. The curves where the metal is to be cut must all be laid out on the surface so that after the plate has been cut and rolled or bent into shape, it will give the desired shape and size to the vessel. The layout should always be made on the surface of the metal that will be the outside of the object and so the plate will have to be bent or rolled down. The piece to be developed is usually imagined to be cut on the shortest element, because it is easier to fabricate in this way.

It is customary to leave about two inches extra material on the edges to be joined, called the lap, to facilitate making the joint, unless it is butt-welded. Table II, Section 7.6, shows which surfaces can be developed. Surfaces which cannot be developed may be made out of sheet steel or plate metal, but only by distorting the metal by heating or pressing it. It should be remembered that on any developed surface all lines are in their true length and all angles are in their true size.

16.6. To Develop a Portion of a Cylinder.

The distance around a cylinder, or its circumference, will be its length when it is developed. This distance must always be measured in the plane of the right section and it will be seen in its true length in the edge view of the cylinder. For this reason, *a cylinder must always be seen in its true length* before it can be developed. It is imagined as being rolled out at right angles to its true length. The elements will always remain parallel to each other, and the distance between them is taken from the view in which they appear as points. With all the elements located, the end of each element may be projected from the true-length view to give the curve desired.

Explanation (see Fig. 605).

It is desired to develop the portion of the cylinder lying above the plane. Since both elevation views show this cylinder in its true length, the development may be made from either one. In this case it is made from the diagonal elevation, because the cut on each element is more easily obtained in this view.

In the development the 16 elements are spaced their true distance apart as seen in the plan, and are drawn parallel to their true length. The end of each element is projected over from its true length to the proper element, as is shown on element 13. The correct developed curve is drawn by connecting the ends of the elements with a fair curved line.

In developing symmetrical objects there may be one or several cuts, thus giving a development in one piece or in several pairs of like pieces. The latter is done in practice to conserve material and labor.

After some experience has been gained, the development may be arbitrarily picked up and placed squarely on the paper, and laid out entirely from divider measurements. However, even then, in developing a cylinder *it should always be imagined as rolling out from a true-length view.* Not less than 16 equally spaced elements should ever be used, and the use of more than this number on larger pieces will insure greater accuracy. Unequally spaced elements may also be used, but they require more work, and are apt to cause errors.

17.6. Practice Problems.

See Chapter VIII, Group 39.

18.6. To Represent an Oblique Cylinder of Revolution Cut by a Level Plane.

ANALYSIS.

The position of the axis of the cylinder will have to be fixed. It is apparent that the cut will show as an ellipse in the plan view. A new elevation view showing the true length of the axis is drawn first, and the extreme elements of the cylinder may be drawn in all views (by Theorem 11). In this new elevation the level plane will appear as an edge, and the cut made across the cylinder by this plane will give the major axis for the elliptical cut in the plan view. The minor axis will be the full diameter of the cylinder. The ellipse may be drawn by using the two axes thus obtained.

Explanation (see Fig. 606).

The axis AB of the cylinder is given in the plan and front elevation views. The level plane is also fixed and contains the point B. The new view 3 shows the axis AB in its true length, and the cylinder is drawn in all three views. The cut of the level

plane across the cylinder in view 3 determines the major axis for the ellipse, and the minor axis is already known. The ellipse is drawn in the plan by use of the two axes and the trammel method, (see Appendix, page 221).

This elliptical cut may also be obtained by drawing the edge view of the cylinder, view 4, selecting elements such as X and Y, and finding where each element is cut by the plane as in Section 13.6. The element method is more basic, but the axis method is

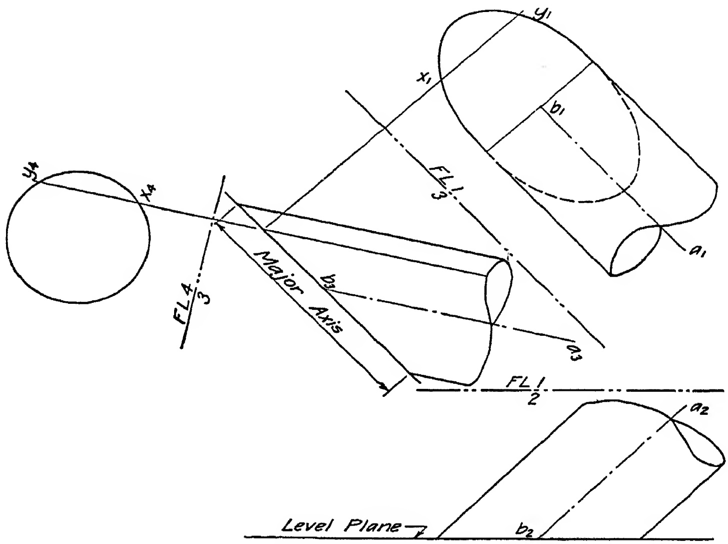


FIG. 606.—Cylinder cut by a level plane.

much faster. After the ellipse has been drawn, it should be checked by projection between plan and front elevation.

If this cylinder is to be developed, it should be rolled out from view 3 and the elements spaced equally in view 4.

19.6. Practice Problems.

See Chapter VIII, Group 40.

20.6. To Represent an Oblique Cylinder of Revolution Cut by a Frontal Plane.

ANALYSIS.

The axis of the cylinder must be given. It is apparent that the cut will show as an ellipse in the front elevation. The solution

will be exactly the same as for the preceding problem, except that the true-length view of the axis will be taken from the front elevation. This is the case because the given cutting plane is parallel to the front image plane and therefore shows in its true

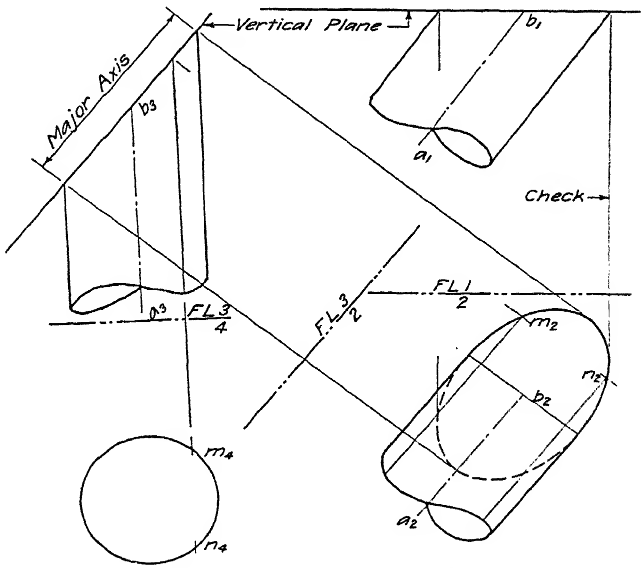


FIG. 607.—Cylinder cut by a frontal plane.

size in the front elevation and as an edge in the true-length view of the cylinder.

Explanation (see Fig. 607).

The axis AB of the cylinder and the vertical plane are given. View 3 shows the true length of the axis AB and the edge view of the vertical or frontal plane passing through the point B. The cylinder is then drawn in all views, and the cut across the cylinder by the plane in view 3 determines the major axis, as in Section 18.6. The ellipse is drawn by the trammel method. It must project between views as before.

View 4 is also shown and is used for solving the problem by the element method. The solution for the elements M and N is shown. The development would be conceived as rolling out the cylinder from view 3.

The most practical method of developing a cylinder of this nature is to square up the paper so that the true length of the cylinder, as in view 3, is perpendicular to the T square. Then the developed surface rolls out parallel to the T square and the elements are all at right angles with the T square. By this method the development may be placed in a square or symmetrical position on a separate sheet of paper.

21.6. Practice Problems.

See Chapter VIII, Group 41.

22.6. To Represent an Oblique Cylinder of Revolution Cut by Any Vertical Plane.

ANALYSIS.

If the vertical plane, as in this case, is not parallel to the front image plane, the major- and minor-axes method cannot be used for obtaining the front view. If the elliptical cut is not in its true size in any view, the major axis will not lie parallel to the axis of the cylinder in that view as it has in the two preceding problems.

The fundamental method of individual elements will have to be used. A true-length view of the axis is drawn and also a view showing the axis as a point. The cylinder is shown in all views. In the edge view, the cylinder is divided up into equally spaced elements, which are then located in every view. The cut on each element is determined in the plan and is projected to the proper element in all other views. The solution is not shown.

23.6. Practice Problems.

See Chapter VIII, Group 42.

24.6. To Represent Any Oblique Cylinder Cut by Any Oblique Plane.

ANALYSIS.

The cylinder and one of its bases will have to be given in both views, as well as the oblique plane. In general, the easiest method of solution is to obtain a new view of both showing the plane as an edge. In this view it will be apparent where each element on the cylinder pierces the plane. The elements may be projected from one view to another by using the given base of the cylinder. However, this process does not place the

cylinder in a position to be developed. A view showing the true length of the cylinder, with both ends of each element, would still have to be drawn, and the cylinder would have to be rolled out from this true-length view.

If the true-length view were to be drawn first, the method of Section 21.3 would have to be used to find where each element of the cylinder pierced the plane. This method would place the cylinder in a better position for developing, but it would cause more work in finding the ends of the elements.

The solution is not shown.

25.6. Practice Problems.

See Chapter VIII, Group 43.

26.6. To Draw a Plane Tangent to a Cylinder from a Point Not on the Surface.

ANALYSIS.

A plane that is tangent to a cylinder will contain one element of the cylinder and will therefore be parallel to all the other elements on the cylinder. This plane will also intersect the plane of the base of the cylinder in a line which is tangent to the base of the cylinder. A line through the given point and parallel to the cylinder will lie in the required plane. From the point where this line pierces the plane of the base of the cylinder a line may be drawn tangent to the base, (on either side). This line and the line containing the given point are two intersecting lines which determine the required plane. There are two possible solutions, giving two different planes.

A good check method for this problem is to draw an edge view of the tangent plane after it has been determined. In this view the plane as an edge must coincide with an extreme element of the cylinder in the same view. The solution is not shown.

27.6. Practice Problems.

See Chapter VIII, Group 44.

28.6. The Cone.

A cone is generated by a straight-line generatrix moving around a straight-line directrix which it intersects. The angle between these two lines may vary.

A cone of revolution is a special type generated when the generatrix revolves about the axis, thus making a constant angle with the axis. The cone of revolution occurs more commonly in engineering practice. It is found in gearing, in roller bearings, and in friction drives in machinery. Cones other than cones of revolution are found in pipe reducers, offsets, and other metal pieces. Since, by definition, the generatrix intersects the axis, all the straight-line elements on a cone pass through the vertex.

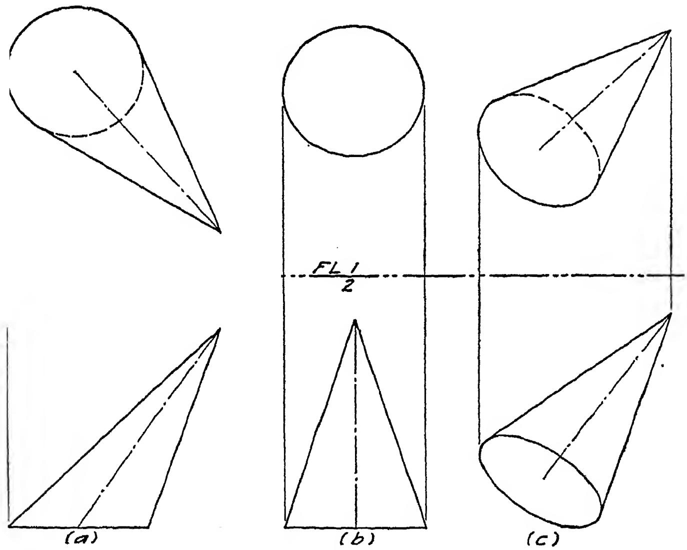


FIG. 608.—Representation of cones.

Every cutting plane which cuts straight-line elements on the cone must contain the vertex of the cone.

A cone is different from a cylinder. It is impossible to draw any view of a cone which will show it as an edge. It is also impossible to draw any one view that will show the true length of all the elements, as can be done with the cylinder. As a rule, the easiest way to solve all cone problems is to use cutting planes giving straight-line elements. As in the cylinder, only one point on the base needs to be determined to locate straight-line elements. The other end of all straight-line elements is the vertex of the cone.

29.6. Representation of a Cone.

A cone is usually represented by showing its vertex, two extreme elements, and some plane section which is called the base. If the vertex is omitted, two plane section cuts must be shown as well as two extreme elements. The cone in Fig. 608 (*a*) is elliptical-shaped. On this cone exactly half the elements are visible in the front view, and more than half are visible in the plan.

When the axis of a cone appears as a point in some view, only the base of the cone is shown in that view, as in the cone of revolution in Fig. 608 (*b*). In this cone all the elements are visible in the plan view, and exactly half the elements are visible in the front view.

Figure 608 (*c*) shows a right cone of revolution with the axis at an angle with both image planes. The base is perpendicular to the axis and is a circle. In the plan view more than half the elements are visible, and in the front view less than half.

30.6. Practice Problems.

See Chapter VIII, Group 45.

31.6. To Find Where a Line Pierces a Cone.

ANALYSIS.

If the given line pierces the cone, it must intersect two straight-line elements on the surface. Both these elements contain the vertex of the cone. A plane may be drawn so as to contain both the given line and the vertex of the cone. If this plane cuts through the cone at all, it will cut across the base and will cut two straight-line elements on the surface of the cone. The base end of these two elements is determined by the intersection of this plane with the plane of the base of the cone. The two elements may then be drawn to the vertex in both views and, since they are in the same plane with the given line, they will intersect that line and give the required piercing points.

Explanation (see Fig. 609).

The line AB and the oblique cone are given in two views. The plane ABV is drawn so as to contain the line AB and the vertex V of the cone. This plane produced is found to intersect the plane of the base of the cone in the line MN, and it actually cuts across the base of the cone at the points X and Y, which are the lower

end of the two required elements. These elements are drawn to the vertex and are found to intersect the line AB at the points P and Q. These two points are on the line AB and on the cone surface, and they are therefore the required piercing points. They may be determined independently in both views and checked by projection.

The solution as given above is completed in the two given views. After the plane ABV is drawn, a new view may be drawn showing that plane as an edge and passing through the vertex of the cone. This plane cuts across the base of the cone

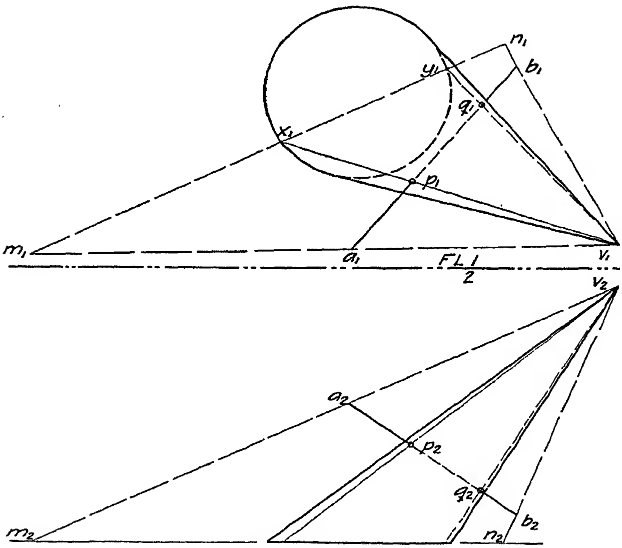


FIG. 609.—Line piercing a cone.

in the same line of intersection, MN, as was obtained by the use of only two views. From this point the procedure will be the same.

32.6. Practice Problems.

See Chapter VIII, Group 46.

33.6. Plane Sections of a Cone of Revolution.

Explanation.

The cone of revolution of Fig. 610 is shown cut across by five different cutting planes.

Plane A, which passes through the vertex, cuts an isosceles triangle. The cuts made by the four other planes are known in mathematics as the conic sections, which were referred to in Section 3.6.

Plane B, being at right angles to the axis of the cone, cuts out a circle.

Plane C, which is not perpendicular to the axis and which cuts all the elements, cuts out a true ellipse.

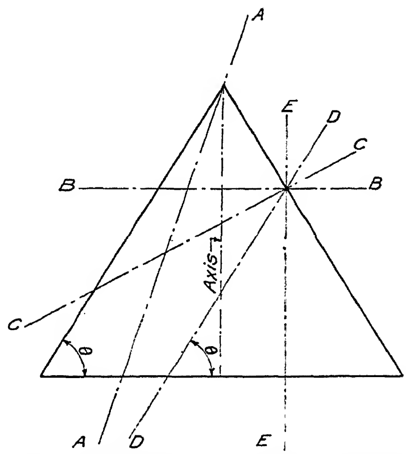


Fig. 610.—Plane sections of a cone of revolution.

Plane D, which is parallel to one element and which has the same slope as this element, cuts out a parabola.

Plane E, which makes any angle greater than θ with the level plane, cuts out a hyperbola.

The true size of each cut may be seen in the view showing the true size of the plane in which it lies. Individual points may be determined by drawing a number of straight-line elements and finding where each element is cut by each plane. It is also possible to determine individual points by using a series of right-section cutting planes which would cut circles from the cone and straight lines from the given planes.

34.6. Practice Problems.

See Chapter VIII, Group 47.

35.6. To Develop a Cone of Revolution.

ANALYSIS.

A cone of revolution is usually developed by using the vertex and some base that is a right section. Since every point on the right-section base is equidistant from the vertex, the locus of all these points in the development will be a portion of a circle with its center at the vertex and with a radius equal to the slant height of the cone. The length of this circle along the arc will

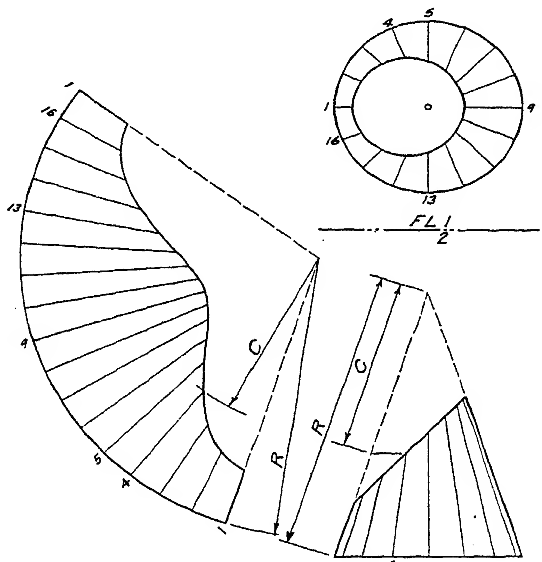


FIG. 611.—Development of a cone of revolution.

be the same as the distance around the base circle. A series of equally spaced elements may be drawn on the cone and also on the development. If the elements happen to be cut off before they reach the vertex, the true length of each element must be found from the vertex (or the base) to the cut end. These lengths, when laid off on the proper elements in the development, establish the points for the desired curve.

Explanation (see Fig. 611).

A portion of a cone of revolution is given with a right-section base and a sloping top. Since the vertex is missing, the cone is

extended so that the vertex can be used. In the plan view the lower base circle is divided into 16 equal spaces, giving equally spaced elements which are drawn to the vertex and numbered in both views. The slant height R is the radius for the developed base circle, and the length of the circular arc is made equal to the sum of the 16 equal spaces around the base circle in the plan. The 16 elements are then drawn in the development. To obtain the curve for the top cut, the true length of each element from the vertex to the top cut is found, in this case by revolution. The figure shows this process for one element (4), whose true length, C, is found by revolution. All the elements of this cone, when revolved, will coincide with the extreme element. Accordingly, it is necessary only to project the upper end of each element to an extreme element to obtain its true length. This true length is then laid off from the vertex along element 4 in the development, to give the point on the upper curve. The ends of all the elements found in this manner determine the top curve. The cone should be cut on the shortest element as shown.

If it had been desired to develop only the portion of the cone above the sloping plane, it would still have been much easier to develop the entire cone to the lower base or to some other right section. This is the easiest way to space the elements in the development, and when they are properly spaced the true lengths to any point on the element are easily determined and laid off.

The method which has just been explained will apply in developing any cone of revolution cut by any oblique plane.

36.6. Practice Problems.

See Chapter VIII, Group 48.

37.6. To Develop Any Cone When the Vertex Is Available on the Drawing.

ANALYSIS.

It is assumed that the given cone is not a cone of revolution. Then the elements all have different lengths, and the base, whether it is a right section or not, will not roll out as a circle. The surface will have to be divided up into elements and the triangulation method used. By this method, the true lengths of two adjacent elements and the true distance between their base ends are laid down in the development to form a triangle.

This is repeated for the adjacent triangle, and so on for the entire surface. This method assumes that the surface between two adjacent elements is a plane surface, which is really not the case. However, with elements taken very close together, the surface becomes so nearly a plane that the error is negligible.

Explanation (see Fig. 612).

The oblique cone is given as shown with a level, circular base. The vertex is available on the paper. The level base is divided

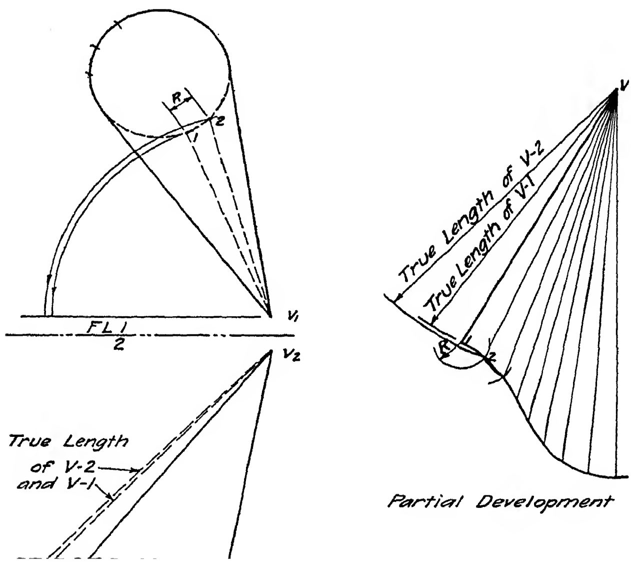


FIG. 612.—Development of an oblique cone by triangulation.

into 16 equal spaces in order to give elements whose lower ends are spaced an equal distance, R, apart. The elements are all drawn to the vertex. The true length[1] of the shortest element, $V1$, is found and is laid down to start the development. The true length of the next adjacent element, $V2$, is then struck off as an arc from V. The point 2 must lie somewhere in the development at a distance equal to the true length of $V2$ from V. But the point 2 must also lie a true distance of R from the point 1. With the distance R for a radius, an arc is drawn with point 1 as a

[1] See Appendix, page 221, for sheet metal workers method of finding true lengths.

center. The point 2 must lie on both of the arcs just drawn, or at their intersection. The entire curve is obtained in the same manner. The figure shows the development of only half of the cone.

The development may be placed in any desired position on the sheet or even on a separate sheet. If it is desired to place the development square with the sheet, the longest element should be laid down first at right angles with the T square. Other elements will then be laid on both sides of the longest one.

If the given cone had been only a partial cone, it would have been produced to its vertex and treated in exactly the same way. The statement of this problem assumes that, if the cone is produced, the vertex will be available within the limits of the drawing paper. The triangulation should be made from the vertex to the base which is farthest from the vertex. The curve for the nearest base may be easily located after the elements are all placed in the development and after the curve for the farthest base has been completed.

38.6. Practice Problems.

See Chapter VIII, Group 49.

39.6. To Develop Any Cone When the Vertex Is Not Available on the Drawing.

ANALYSIS.

Since the vertex is not available on the drawing, the cone will have to be given with an upper and a lower base. The conical surface is assumed to be composed of a series of trapezoidal areas which, if they could be produced to the vertex, would be triangles as in the previous problem. These areas lie between two adjacent elements and a section of each of the upper and lower bases. A diagonal is drawn across this trapezoidal area to divide it into two triangular areas. True sizes of these triangles are then found and laid down in consecutive order to obtain the development.

Explanation (see Fig. 613).

The cone is given as shown, having both upper and lower bases level. The surface is first bisected by a vertical plane containing the axis of the cone. This plane locates the element BC and the two points B and C from which each base is divided into the same number of equal parts. The corresponding elements are then drawn between the two bases, as the element AD. These

elements will all meet at the vertex in space. They give the first elemental trapezoidal area, $ABCD$, to be developed. The diagonal is drawn from A to C. The triangle ABC is then developed exactly as in Section 37.6. The triangle ACD is developed next, and the rest of the surface in the same way. The radii N and M are the true lengths of AC and AD, respectively. The radii R and S are the true lengths of the dis-

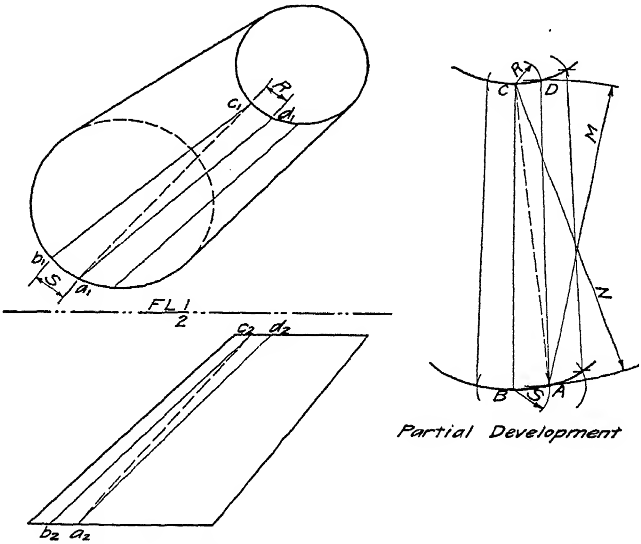


FIG. 613.—Development of a cone without using the vertex.

tances between equally spaced elements on the upper and lower bases respectively. See Section A2 in Appendix for special method to use here for finding true lengths.

Again the assumption has to be made that the trapezoid and the triangle are plane surfaces, which is not quite true. However, it affords an approximation sufficiently close for all practical purposes.

This problem would be more easily solved by the method of Section 37.6 if the surface could be produced to a vertex. In some cases it is impossible to get the vertex within the limits of the paper or the available steel plate, and then the method given in this section must be used.

40.6. Practice Problems.

See Chapter VIII, Group 50.

41.6. To Draw a Plane Which Is Tangent to a Cone and Contains a Given Point Not on the Surface of the Cone.

ANALYSIS.

A plane that is tangent to a cone will contain one straight-line element on the cone, and therefore it will contain the vertex as well. Since it must also contain the given point, it will contain a line from the given point to the vertex. The intersection of this tangent plane with the plane of the base will be a line which is tangent to the base of the cone. The point at which the line connecting the given point with the vertex pierces the plane of the base is one point on the line of intersection which is tangent to the base. From this point the line of intersection may be drawn tangent to the base of the cone (on either side), giving two possible solutions. The required plane is therefore represented on the drawing by its intersection with the plane of the base of the cone and the line from the given point to the vertex. The solution is not shown here.

42.6. Practice Problems.

See Chapter VIII, Group 51.

43.6. The Convolute.

The convolute is a single curved surface which is generated by a straight line moving so as to remain tangent to any double curved line.

The helical convolute is a convolute that is generated when the double curved line is a helix.

The generating line is sometimes assumed to extend (in all its different positions) to a given level plane and sometimes it is assumed to remain a constant length. Both these conditions are illustrated in Fig. 614.

The most practical application of the convolute is the helical convolute which is found in the blade of a screw conveyor. In this case the tangents to the helix do not extend to a level plane but only to a larger concentric cylinder inside of which the blade is to move.

Mathematicians call this surface a developable helicoid, a designation which is not strictly correct. The helicoid and the convolute are very similar, but Table II shows them to be in entirely different classes.

The convolute is a developable surface, but only within certain limits, which are determined by the slope of the helix. If this slope is exactly 45 degrees, the convolute can be developed in one piece for only 1.41 turns around the cylinder. For angles

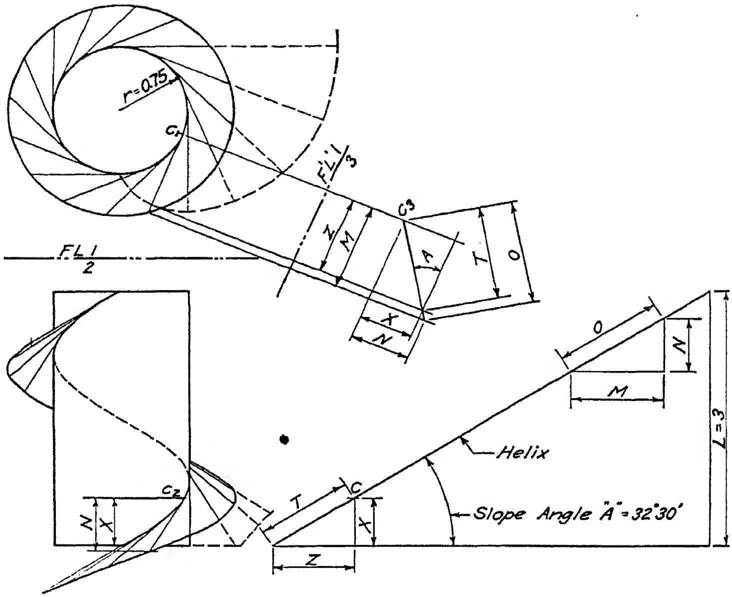


FIG. 614.—Convolute.

of slope less than 45 degrees, it can be developed from 1.0 to 1.41 turns about the cylinder in one piece. For angles of slope between 45 and 90 degrees, the number of turns which can be developed varies from 1.41 to an infinite number. The number of turns it is possible to develop any helical convolute may be expressed in the form of an equation as follows:

$$N = \frac{1}{\cos A},$$

where N is the number of possible turns and A is the true-slope angle of the helix in degrees (see Section 5.6, Fig. 601).

44.6. Representation of a Helical Convolute.

Explanation (see Fig. 614).

The helical convolute is represented in a drawing by showing the helix and its cylinder, and several positions of the tangent to the helix. In Fig. 614 the given helix is drawn as in Section 5.6, by means of 16 equally spaced points. Figure 614 also shows the helix unwrapped from the cylinder in order to show its true length and true slope and to find the length of the tangents in the plan view. The solid curved lines show the convolute extending only to a larger concentric cylinder and for one complete turn around the cylinder. The dash lines show (only partially in each view) where the convolute would be if it extended to the level plane of the base of the cylinder.

To draw the convolute, at any point of the helix, such as C, a tangent is drawn to the helix. In the plan this is drawn tangent to the cylinder. If it is to extend only to the larger cylinder, the plan-view length is immediately seen to be equal to M. If this distance is laid off level under the unwrapped helix, the difference in elevation between the ends of the tangent is seen to be equal to N. This fixes the front view of this tangent, and all other positions in this view are obtained by using this constant difference in elevation.

If it is desired to show the tangent extending to the level plane, it is apparent in the front view that the tangent would meet that plane at an elevation of X below point C. Projecting C over to the unwrapped helix fixes the plan view of the tangent as a distance Z in length, because the slope of the tangent is the same as the slope of the unwrapped helix.

This is shown a little clearer in view 3, which shows the tangent meeting the level plane when its length is T and going on through to the outer cylinder when its length is 0. This view shows the tangent in its true slope, and true length, the plan-view length, and gives the difference in elevation between the ends for both conditions which have been illustrated.

45.6. Development of the Helical Convolute of Figure 614.

Explanation (see Fig. 615).

It is possible to develop a convolute by assuming it is composed of a series of triangles and by laying down the true size of these triangles in successive order. This is a cumbersome method, and there is another method which is very much simpler.

Since the helix has a constant curvature and a constant slope it will be a circle in the development of the convolute with a radius R, which is larger than the radius r of its cylinder. It has been proved by mathematics that the radius of this developed circle is equal to the radius of the helix cylinder divided by the square of the cosine of the slope angle of the helix.

$$R = \frac{r}{\cos^2}$$

Using this equation, the radius of the developed helix circle is calculated to be 1.053 and the circle is drawn. The distance

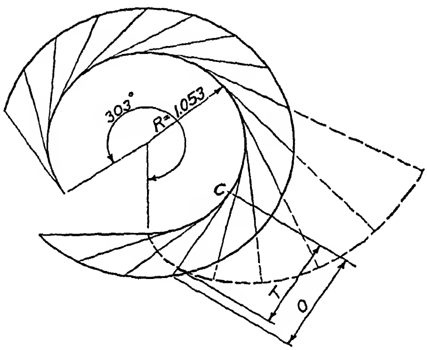


FIG. 615.—Development of a convolute.

the helix extends along this circular arc is equal to its slope length, or true length. This true length of the unwrapped helix is divided into 16 equal parts, and these parts are stepped off around the circle. At each of the division points on the circle a tangent is drawn, as at C, and is laid off in its true length (either T or O) from the helix. The ends of the tangents give the developed curve for either condition.

If the screw-conveyor surface as limited by the larger cylinder in Fig. 614 is to be developed, only one tangent needs to be drawn and laid off in its true length. The outside end of this tangent will determine the radius of the outside helix circle. This radius may also be calculated by the formula given above.

In practice the convolute will always be a surface limited by two concentric cylinders. The use of the formula greatly simplifies this development. The problem which has just been explained has assumed only one turn around the cylinder. The

development shows that it is possible to develop a little more than one turn. The exact number of turns for the convolute shown, as calculated by the formula in Section 43.6, is found to be 1.186. It can also be easily calculated that one turn around the cylinder developed to 303 degrees of the circle in the flat. This result is found by multiplying 360 degrees by the cosine of the slope angle of the helix.

Attention is called to the fact that, theoretically, the convolute surface is the only one that can actually be developed without distortion for the purpose of making a screw-conveyor blade. However, this surface does not give a blade which is perpendicular to the shaft. Such a blade is to be preferred in practice, and the method for obtaining it will be explained in the next section.

46.6. Practice Problems.

See Chapter VIII, Group 52.

47.6. The Helicoid.

The helicoid is a warped surface which is generated by a straight line moving about an axis in such a way that any two points on the line describe concentric helices having the same lead. The generating line need not intersect the axis, although it nearly always does. When it intersects the axis it also makes with it a constant angle. If this angle is equal to 90 degrees, the surface is a right helicoid. Examples are the wearing surface of a square screw thread, a winding ramp (or chute) of constant grade, or a screw-conveyor blade. If the angle with the axis is an acute angle, the surface is an oblique helicoid, which would be found on the surface of a V-thread screw. Figure 616 shows a partial elevation of a cylindrical tower with a winding ramp and hand-rail going up around the outside. The ramp surface is a right helicoid.

The helicoid is represented by showing several positions of the generatrix and the helix with its axis. The length of the generatrix is limited, in all practical applications, by the given helix and by another larger or smaller concentric helix which will be fixed by the conditions of the problem. These two limiting helices are shown in Fig. 616 together with a few positions of the generatrix. The screw-thread surfaces are so common that they are not shown.

Theoretically, it is impossible to develop a helicoid. Wherever this surface occurs in practice, made of steel plate or concrete,

it must be made by distorting the metal or by approximating the surface by dividing it up into small sections. Most screw-conveyor blades are actually built by making an approximate development of a right helicoid, since the lead is usually comparatively small. The radius R of the developed inside helix may be calculated by the formula $R = r/\cos^2 A$ (see Sec. 45.6), and thus the developed helix circle may be drawn. The true

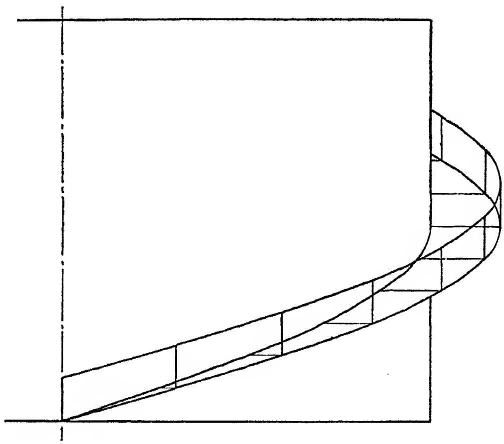


FIG. 616.—Helicoid.

length of this helix for one complete turn may be determined graphically (see Fig. 601) or by calculation, and this length laid out on the developed helix, which is the arc of a circle. The outer helix may be developed in the same way and the plate cut at both ends on radial lines, forming a segment of a circle. In shop practice it has been found that these radial lines will have to be trimmed or sheared off just a trifle to make a good fit. Also the plate will have to be hammered to make it come perpendicular to the axis, as a right helicoid should. This method of building a screw-conveyor blade is very similar to the convolute method of Sec. 45.6. It is usually followed in practice because of the simple method of laying out the blade and because of the desirability of having the blade surface square with the axis.

48.6. Practice Problems.

See Chapter VIII, Group 53.

49.6. Hyperbolic Paraboloid.

The hyperbolic paraboloid is a warped and a double-ruled surface which has two straight lines and one plane for directrices. The straight-line generatrix moves so that it constantly touches two nonparallel, nonintersecting straight lines and remains parallel to some plane. It has the appearance of a plane that has been twisted.

The surface is represented by showing the two straight-line directrices and several positions of the generatrix.

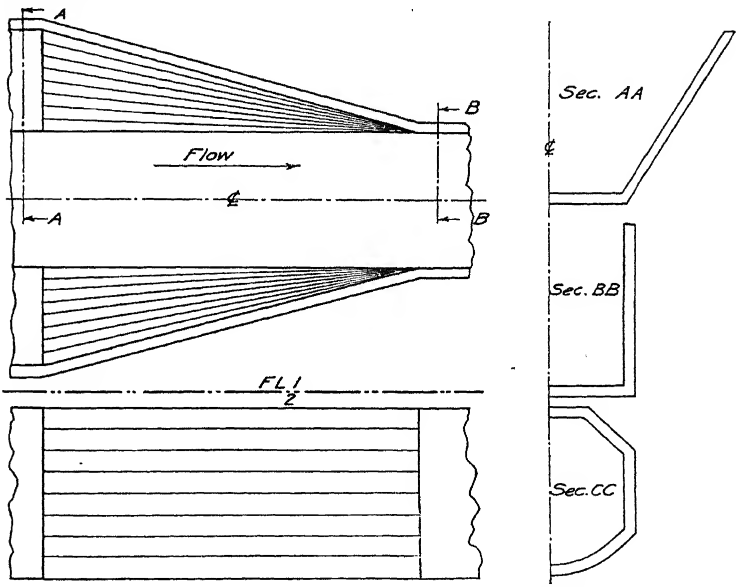


Fig. 617.—Hyperbolic paraboloid. Transition.

The hyperbolic paraboloid has recently become a rather common surface in concrete construction. As carpenters have become more expert in building forms for concrete construction, engineers have felt more free to submit plans calling for warped surfaces if these will give either a better appearance to the structure or a higher operating efficiency. Consequently, we often find this surface on concrete dams, irrigation ditches, tunnels, wing walls, piers, etc.

In certain metal offset pieces this surface is approximated, or really avoided entirely, by connecting the opposite ends of the

straight-line directrices with a diagonal straight line. This divides into two triangular-shaped planes the surface which otherwise would be a hyperbolic paraboloid.

In Fig. 617 is shown a very practical illustration of the occurrence of this surface in irrigation work. The open ditch of section *AA* must be gradually changed into a syphon having a closed section *CC* in order to conduct the water down into a deep ravine and up to the proper elevation again on the far side of the ravine. The section *AA* is first changed to the square section *BB* and then to the section *CC*. The portion between the sections *AA* and *BB* is shown in Fig. 617. It is called a transition, and its sloping sides are hyperbolic paraboloids. Between sections *BB* and *CC* there would be more surfaces of this same kind. The plane directrix for the surface shown is a level plane, and the two linear directrices are the vertical and the sloping lines at the ends of the surface.

The hyperbolic paraboloid is an undevelopable surface, and it is impossible to make a form for this surface in one piece. The forms are usually made by using narrow and thin pieces of lumber fastened to the straight-line directrices at each end and supported by intermediate straight-line supports, since the surface is double-ruled. These strips give a very fair surface. A strip skiff is built in somewhat the same manner.

Any vertical plane cutting across this surface and parallel to both linear directrices will cut straight-line elements. Any level plane, or any plane that is parallel to the plane directrix, will also cut straight-line elements. Any other plane cutting across this surface will intersect it in curved lines.

50.6. Practice Problems.

See Chapter VIII, Group 54.

51.6. Conoid.

The conoid is a warped surface which is generated by a straight line moving so that it touches one straight-line directrix and one curved-line directrix and remains parallel to some plane directrix. The two linear directrices must not lie in the same plane. The curved-line directrix may be either an open or a closed curve.

This surface is represented by showing both linear directrices and at least the extreme positions of the generatrix, in all views. Sometimes several other positions of the generatrix are shown.

Figure 618 shows three views of a conoid which has a vertical plane parallel to the front image plane for its plane directrix. Its curved directrix is a circle, in this case, and its straight-line directrix is the level line AC. Several random positions of the generatrix are shown in each view.

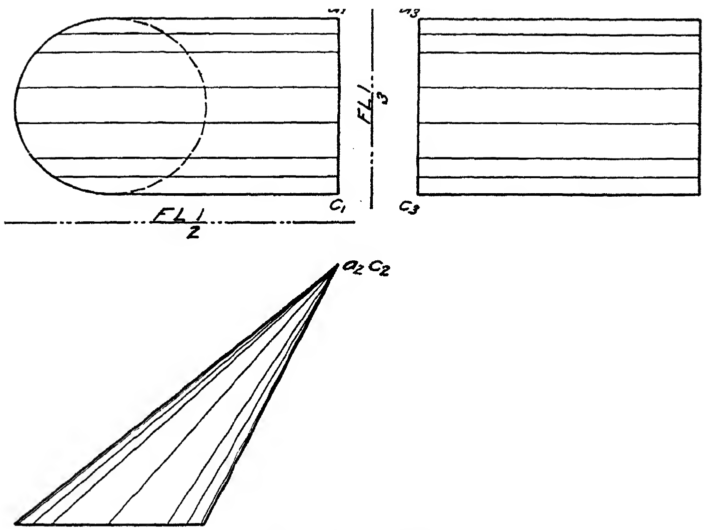


Fig. 618.—Conoid.

The conoid has very few uses in engineering practice. It is an undevelopable surface.

52.6. Practice Problems.

See Chapter VIII, Group 55.

53.6. Cylindroid.

The cylindroid is a warped surface which is generated by a straight line moving so that it touches two curved-line directrices and remains parallel to a plane directrix. The two curved-line directrices must not lie in the same plane. They may be either closed or open curves.

The cylindroid is represented by showing the two curved-line directrices and several positions of the generatrix. Figure 619 shows two views of a cylindroid having a level plane directrix. In order to show this surface more clearly the several positions

of the generatrix are shown as solid elements on the upper face and as dashed elements on the under side of the surface. The twist or warp of this surface is clearly indicated in the plan view.

The cylindroid, like the conoid, has very few direct applications in engineering practice. However, these two surfaces, together

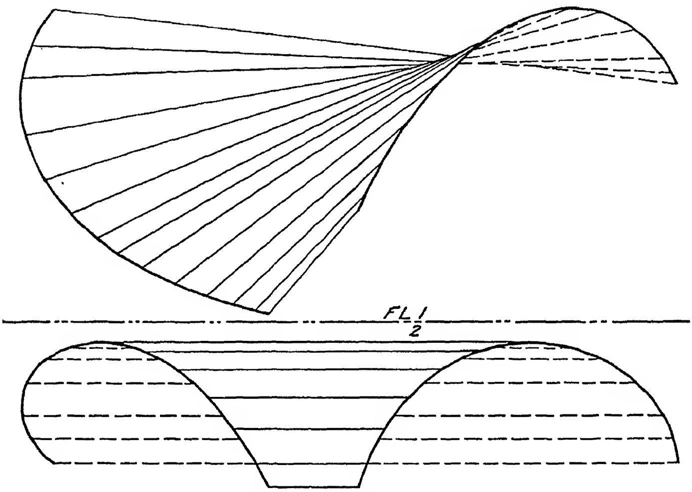


Fig. 619.—Cylindroid.

with the hyperbolic paraboloid, occur in naval architecture on ship bodies and also on airplane fuselages. Nearly every stream-lined surface will be found to be one of these three surfaces or some combination of them.

The cylindroid is also an undevelopable surface.

54.6. Practice Problems.

See Chapter VIII, Group 56.

55.6. Special Cases and Limitations.

The hyperbolic paraboloid, the conoid, and the cylindroid, which have just been explained, are so closely related to each other that it is sometimes difficult to classify them. Each surface has two linear directrices and one plane directrix to govern the motion of the generatrix. The definition of each surface prescribes the nature of the guides which control the

travel of the straight line in generating that surface. And yet, a generatrix moving according to the definition for a conoid might also generate a hyperbolic paraboloid at the same time. To prove this last statement, a curved line may be assumed to lie somewhere on a hyperbolic paraboloid. If a generating line were to follow this curved line and one of the straight-line directrices, it would generate both a conoid and a hyperbolic paraboloid. In the same way it can be shown that a generatrix could move so as to satisfy the definition for a cylindroid yet could generate a conoid at the same time. These are possible cases, but they would occur only when the linear directrices had certain special shapes and certain relative positions.

It is easy to assume the two linear directrices in such positions that it is impossible for the generatrix to touch them both and remain parallel to the plane directrix. All three directrices can not be fixed arbitrarily. Even if only the linear directrices were fixed in space, it might be impossible to determine the plane directrix so as to generate the desired surface and traverse the entire length of both the linear directrices.

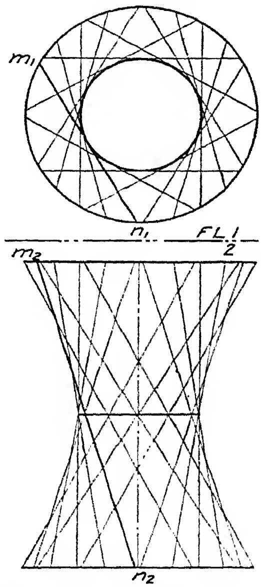


Fig. 620.—Hyperboloid of revolution of one sheet.

The extent of these three surfaces, and therefore their use, is very much limited. In practice, the desired surface is generated as far as possible, and a new surface of the same kind, or even a surface of an entirely different nature, may have to be used to complete the desired piece.

56.6. Hyperboloid of Revolution of One Sheet.

The hyperboloid of revolution of one sheet (or of one piece) is a warped surface which is generated by revolving one straight line about another straight line as an axis, provided these two lines are not parallel or intersecting.

Any plane section of this surface which contains the axis will be a hyperbola, and any plane section which is at right angles

to the axis will be a circle. The smallest possible circular section is called "the circle of the gorge," and it will have a radius equal to the shortest distance between the two lines.

This surface is usually represented by showing the axis, several positions of the line which revolves, the two end or base circles, and the circle of the gorge. Figure 620 shows two views of a hyperboloid of revolution which is generated by revolving the line *MN* around the vertical axis.

This surface cannot be developed.

57.6. Practice Problems.

See Chapter VIII, Group 57.

58.6. Sphere.

A sphere is a double curved surface that is generated by revolving a circle about a line passing through its center. A sphere

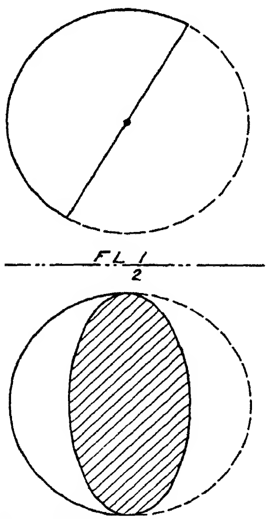


Fig. 621.—Great circle of sphere.

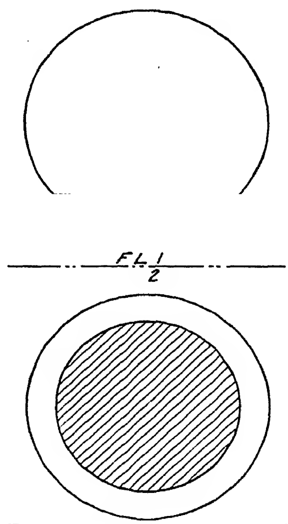


Fig. 622.—Small circle of sphere.

will appear exactly the same in all orthographic views. It always appears to be the true size of the circle by which it is generated. It is therefore represented in any view by merely showing its generating circle.

Any plane section through a sphere at any angle will be a circle. If the cutting plane contains the center of the sphere, the section is called a "great circle"; if it does not contain the center of the sphere, the section is said to be a "small circle."

Figure 621 shows two views of a sphere which is cut by a plane passing through its center. The circle cut out is a great circle which appears as an ellipse in the front view. Figure 622 shows the same sphere cut by any plane which does not contain its center. The circle cut by this plane is seen to be a small circle, and its center lies on a line which is perpendicular to the cutting plane and passes through the center of the sphere.

The earth is practically a sphere, and those who study navigation must be familiar with great and small circle routes. The domes of some buildings are portions of spheres, and many roller bearings depend upon spherical balls for their operation. Also some steel water tanks are made with a hemispherical bottom, and gas retorts often have a top which is partly spherical.

59.6. To Locate a Point on a Sphere, Having Given One View.

If one view of a point is given as being on the surface of a sphere, a small-circle element may be drawn containing the point. This element should be drawn so that it is a straight line in one view and a circle in the other. The point will have to be on this element in both views. A large-circle element could also be used, but might require an extra view to show it as a circle. The construction is not shown here.

60.6. To Find Where a Line Pierces a Sphere.

A plane is drawn so as to contain the given line and either a great or a small circle on the sphere. A view showing the true size of that plane will show the two points where the given line intersects the circle which is cut out by the plane. These two points are on both the line and the surface of the sphere, and they are therefore the desired piercing points. The solution is not shown.

61.6. To Draw a Plane Tangent to a Sphere and Containing a Given Line.

A new view is drawn showing the sphere and also showing the line appearing as a point. Any plane containing the given line will show as an edge in this view. The required plane is so

drawn that it contains the line (which is a point in this view) and consequently is tangent to the sphere, on either side. Two solutions are possible. The tangent plane may be located in any other views by selecting any two random lines upon it and projecting them to the views desired. The solution is not shown.

62.6. Practice Problems.

See Chapter VIII, Group 58.

63.6. Approximate Development of a Sphere.

The sphere itself is an undevelopable surface. It is sometimes necessary to construct a spherical surface from steel plate or other material, and then an approximate method must be used.

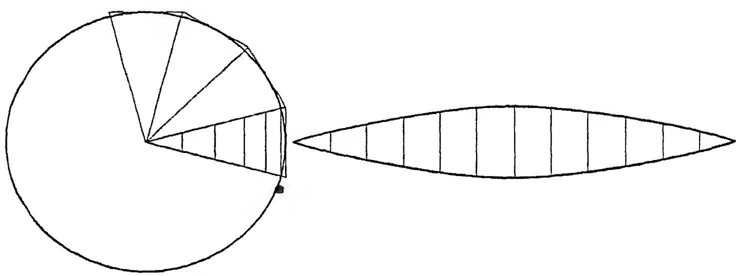


FIG. 623.—Meridian method of developing a sphere.

Two methods are in use, both of which consist in breaking up the surface of the sphere into small portions and assuming each portion to be a part of a surface which can be developed.

The meridian method, shown in Fig. 623, consists in cutting up the surface by a series of planes passing through the center of the sphere and assuming that each small section is a portion of a cylinder. This small portion is easily developed, as is shown in the drawing. If the planes are spaced an equal distance, or angle, apart around the sphere, it is necessary to develop only one piece, for all the cut sections are alike. This is known in practice as the orange-peel method.

The zone method, shown in Fig. 624, consists in cutting the sphere by a series of parallel planes and assuming that each section of the surface cut is a portion of a cone of revolution. Each of these sections has to be developed independently, for they all lie on cones of different size, that is, for half a sphere.

Two of the cones are shown in the drawing, and their development is also shown.

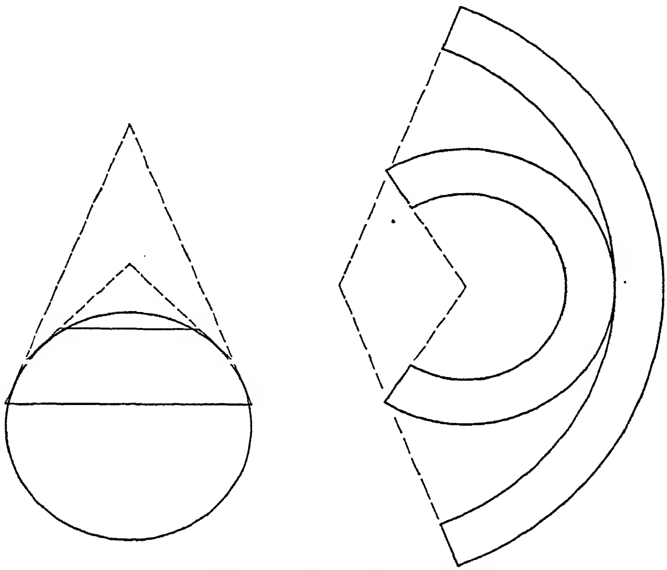


FIG. 624.—Zone method of developing a sphere.

In using either of these methods the surface should be divided up into as many sections as possible in order to approach more closely the actual spherical surface.

64.6. Torus or Annulus.

The torus is a double-curved surface generated by revolving a circle about a straight-line axis which does not contain the center of the circle. If this axis does not intersect the circle, an open torus is generated with a hole through its center. A torus is represented by showing just the extreme projecting elements in each view (see Fig. 625).

Elbows furnish nearly all the practical examples of a torus. An elbow is usually a quarter of an approximate torus. The shop method of making one of these steel elbows is to make a series of diagonal cuts, by means of a torch, across a stock steel pipe. Adjacent cuts are reversed in direction, thus giving

trapezoidal-shaped sections of pipe. Every other section of pipe is then reversed, the long sides all being turned on the same side to make the elbow curve. After a little necessary chipping and fitting the sections are then welded together.

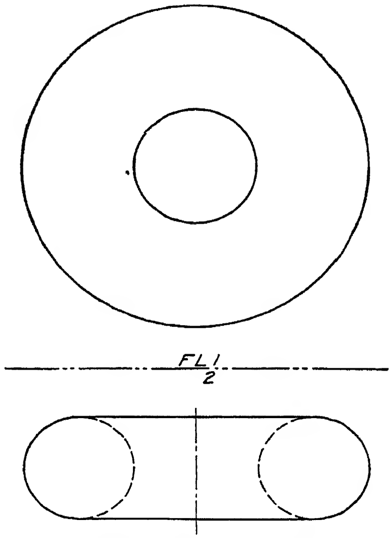


FIG. 625.—Torus or annulus.

65.6. Practice Problems.

See Chapter VIII, Group 59.

66.6. Ellipsoid of Revolution.

The ellipsoid of revolution is a double-curved surface that is generated by revolving an ellipse about one of its rectangular axes. If it is revolved about its major axis, the surface generated is a prolate ellipsoid. It has the appearance of a football. If the ellipse is revolved about its minor axis the surface generated is an oblate ellipsoid. It has the appearance of a discus or a door knob. Tanks of this shape are now used for large storage tanks for gas.

A prolate ellipsoid will appear as an ellipse in two views and in the third view as a circle having a diameter equal to the minor axis.

An oblate ellipsoid will appear as an ellipse in two views and in the third view as a circle having a diameter equal to the major axis.

67.6. Paraboloid of Revolution.

The paraboloid of revolution is a double-curved surface that is generated by revolving a parabola about the symmetrical axis which passes through its focus. In a three-view drawing of this surface, two views will show it as a parabola, and the third view will show it as a circle.

This surface occurs often in searchlight reflectors, in modified form, because of its peculiar properties in reflecting parallel rays. If it is made out of sheet metal it must be pressed into shape, for it is undevelopable.

68.6. Hyperboloid of Revolution of Two Sheets.

The hyperboloid of revolution of two sheets is a double-curved surface which is generated by revolving a hyperbola about an axis containing both its foci. It must be remembered that the mathematical equation for a hyperbola places the curve in two opposite quadrants and that the two curves are entirely separate. Therefore this surface is said to have two sheets, because it is in two separate and distinct pieces. It must be distinguished from the hyperboloid of revolution of one sheet in Section 56.6. In a three-view drawing of this surface, two views would show it as two separate hyperbolas and the third view would show it as a circle.

69.6. Practice Problems.

See Chapter VIII, Group 60.

70.6. Miscellaneous.

All of the double-curved surfaces that have been introduced here have been surfaces of revolution, because these occur more frequently and because their generation can be more easily defined. A countless number of other double-curved surfaces might be generated by complex motions of curved lines. Most of these surfaces do not even have a name and seldom occur in engineering practice. They have been omitted for this reason.

A sufficient number of surfaces have been explained to enable the reader to classify practically every surface with which he will

ever come in contact in actual experience. He would know what kind of elements were on the surface, he could represent the surface, solve whatever problems were necessary in connection with it, and make an approximate development of it if necessary. A few of the more common cases of intersections of the various surfaces of this chapter will be considered in the chapter following.

CHAPTER VII

INTERSECTION OF SURFACES

1.7. Introduction.

Occasionally in engineering construction, and often in large special pipe fittings, an object is made up of two different surfaces that intersect each other, each surface extending only to its line of intersection. For development work this line of intersection must be accurately determined, which means that it will have to be located accurately in two orthographic views. A large variety of problems might be introduced in this chapter, corresponding to all the different possible combinations of the surfaces which were discussed in Chapter VI. However, it is intended to give only a very few representative problems which may occur most frequently in engineering practice. In these problems methods of solution will be introduced which are quite typical as far as the space analysis is concerned. A general procedure will then be given containing a few rules and suggestions to be followed in the solution of miscellaneous intersection problems.

2.7. Two Plane Surfaces. Prisms and Pyramids.

The intersection of objects which are bounded entirely by plane surfaces may be determined completely by the methods of intersection of planes. These methods have already been explained in Sections 20.3 and 21.3.

3.7. Plane Surface and Any Other Surface.

First Method.

A new view of both surfaces in which the plane appears as an edge will show where each element of the other surface intersects the plane, whether the elements used are straight lines or curved. Each intersection point may then be projected to the proper element in the desired views.

Second Method.

If the nonplanar surface has straight-line elements, the points where these elements pierce the plane may be found by using the method of two views only, Section 18.3. A series of these points, if connected with a fair curve, will give the entire line of intersection.

Third Method.

If the nonplanar surface has circular elements for its simplest ones, the better method is to intersect both the given surfaces with a third plane, which will cut out a circular element and a straight-line element from the two given surfaces. It will be necessary to use a series of these planes in order to determine a sufficient number of points.

4.7. Two Cylinders with Their Bases in the Same Plane.

First Method.

The points at which the straight-line elements on one cylinder pierce the other cylinder are points on both cylinders; that is, on the line of intersection of the two cylinders. These points may be determined by using either method of Section 11.6.

Second Method.

A series of planes could be passed cutting circular or elliptical elements on both cylinders. Since these two elements would lie in the same plane, they must intersect and give points on the required line of intersection or miss each other altogether. This method should be used only when circular elements are cut from both cylinders.

Third Method.

ANALYSIS.

A plane may be passed so that it is parallel to both cylinders and therefore will cut straight-line elements on both. These elements, two on each cylinder, lie in the same plane, and they must intersect (or be parallel), giving four points on the line of intersection. Other planes, all parallel to the first one, are passed to determine other points on the desired curve.

Explanation (see Fig. 701).

The two oblique cylinders are given in the position shown. From any random point, such as X, two lines are drawn parallel to these two cylinders. These lines determine a plane which is parallel to both cylinders and which will cut straight-line elements on both (if it intersects them). This plane intersects the plane of the bases in the line MN, which line determines the two elements the plane cuts out of each cylinder. These four elements are shown in both views; the four points at which they

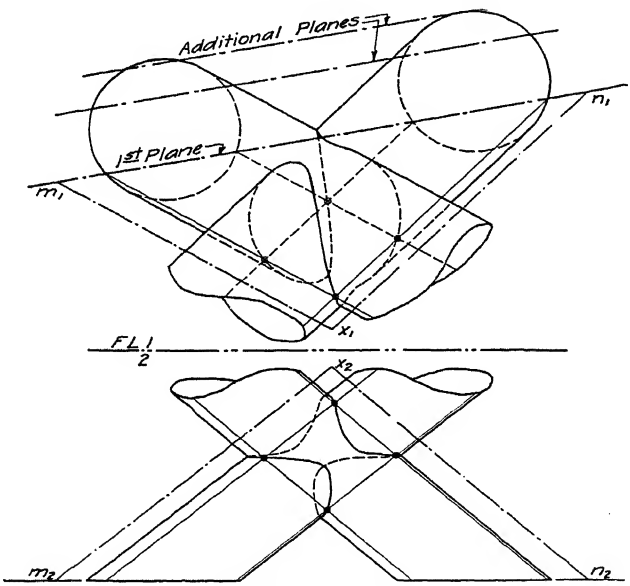


Fig. 701.—Two cylinders. Bases in the same plane.

intersect are on the line of intersection of the two cylinders. These points are determined independently in each view, and they should be checked by projection between views. A series of planes taken parallel to this first plane will establish sufficient points to determine the entire line of intersection.

These cutting planes, although they are taken at random, should be drawn first, so that they will contain the extreme elements on the cylinders in both views. This may require eight planes, and they will determine the exact tangent points at which the curve touches all the extreme elements. After these eight tangent points have been determined, other planes are taken

where they are needed to complete the curve. In order for a point on a line of intersection to be visible, it must lie on a visible element on both surfaces. This method may also be used in finding the line of intersection of two prisms, or a prism and a cylinder.

5.7. Two cylinders with Their Bases in Different and Non-parallel Planes.

First Method.

The first method is the same as the first method of Section 4.7. It applies equally well regardless of the position of the bases.

Second Method.

ANALYSIS.

The axes of the two cylinders would not occupy any different relative positions in space from those they would for the third method of Section 4.7. The cylinders themselves are just cut off by different planes. Therefore the space analysis remains just the same, and the same method of using cutting planes will apply. The only difference in the detail solution is that the assumed cutting plane will now have to intersect the two different planes in which the bases lie. Additional cutting planes will all be parallel to the first plane drawn, which is parallel to both cylinders.

Explanation (see Fig. 702).

The two cylinders are given in the position shown. The plane AXZ is determined, parallel to both cylinders. This cutting plane is found to intersect the plane of the base of the B cylinder in the line MN, the line RN being used to determine the point N. The front elevation shows MN intersecting the base of this cylinder and determining the two elements of the cylinder which are cut out by this plane. The same cutting plane AXZ is found to intersect the plane of the base of the A cylinder in the line AK, by use of the auxiliary line XY on the plane. In the side elevation this line AK is seen intersecting the base of the A cylinder and determining the two elements which are cut from this cylinder. The two elements on each cylinder are now located in all the views by projection. All four elements lie in the plane AXZ, and therefore they intersect at the points shown, giving four points on the curve of intersection.

The two planes containing the cylinder bases form a dihedral angle with its vertex in the vertical line HE. Any plane cutting across this dihedral angle must cut out a plane angle with its vertex on the line HE. In other words, the two intersection lines

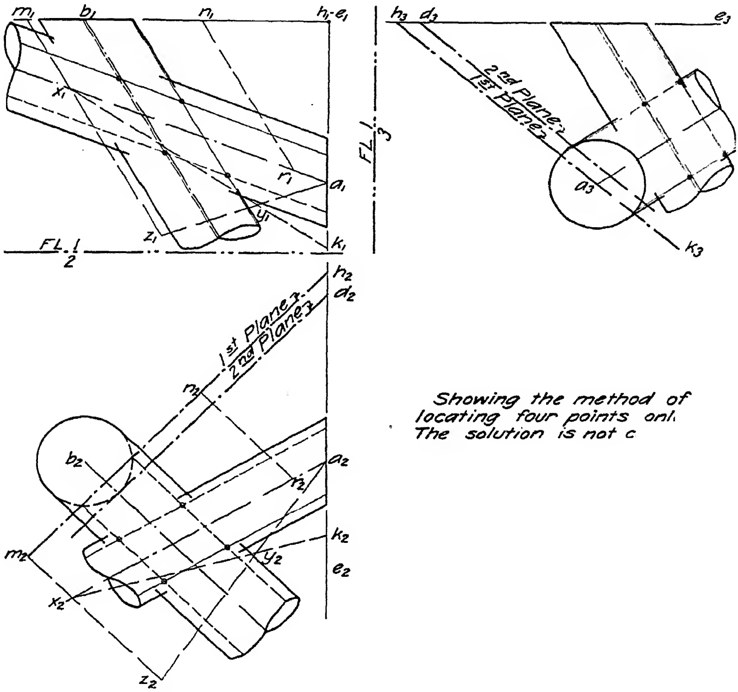


FIG. 702.—Two cylinders. Bases in different and nonparallel planes.

MN and AK are the sides of this plane angle, and they must meet on the line HE. They are seen to check by meeting at the point H. This fact is used in locating additional cutting planes. The second plane is now assumed, and in the front elevation its intersection is drawn at random, either tangent to or intersecting the base of the B cylinder. This intersection with the base plane meets the line HE at D. The point D is then located on HE in the side elevation, giving d_3. From d_3 the intersection with the plane of the base of the A cylinder is drawn parallel to the intersection of the first plane. In Fig. 702 the second plane is drawn tangent to the B cylinder, to show that it is the lowest plane that may be used.

Other planes are drawn in this same way until the special tangent points are determined and the entire curve is established. The solution is not complete but is shown just far enough to make the method clear. If the points on the curve are determined in all views by the actual intersection of elements they should be checked to see whether they project or measure accurately between views.

The method just described is a general one which will apply to all cylinders regardless of their shape or position in space. It may also be used to find the intersection of two prisms or the intersection of one prism with one cylinder.

6.7. Practice Problems.

See Chapter VIII, Group 61.

7.7. Two Cones with Their Bases in the Same Plane.

First Method.

By the method of Section 31.6, the two points at which any straight-line element on one cone pierces the other cone may be determined. The entire curve of intersection may be found by repeating this process with a sufficient number of elements. This is a rather laborious method, and it does not establish special points on the curve with sufficient accuracy.

Second Method.

Both cones may be cut by a series of planes parallel to their bases. The elements cut from the cones will probably be circles or ellipses. The intersections of these elements which lie in the same plane will be points on the desired line of intersection. The only condition which would make it practicable to use this method would occur when the planes cut circular elements on both cones. Even then the special tangent points on the curve are found only by a cut-and-try method.

Third Method.

ANALYSIS.

A cone has straight-line elements, all of which pass through its vertex. Therefore every cutting plane which cuts straight-line elements on the cone must contain the vertex of the cone. Also,

for the same cutting plane to cut straight-line elements on two cones, it must contain the vertices of both cones. It must also contain a straight line connecting these two vertices. If this straight line is drawn and produced until it pierces the plane containing the bases of both cones, this piercing point will also have to lie in every cutting plane. If all the cutting planes are

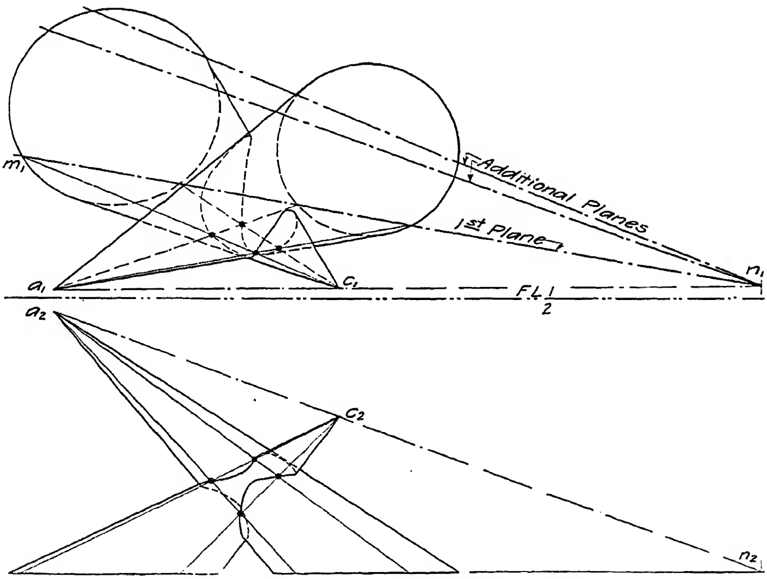


FIG. 703.—Two cones. Bases in the same plane.

produced until they intersect the plane of the bases, these intersections will all meet at this piercing point of the line of the vertices with the plane of the bases. These intersections may be drawn in at random, so long as they cut across the bases of both cones, and the points where they intersect the bases determine the elements which will intersect each other.

Explanation (see Fig. 703).

The two oblique cones are given as shown. The line AC, which contains both vertices, pierces the plane of the bases at the point N. The first cutting plane is drawn at random, its intersection with the plane of the bases being the line MN. This line is seen to cut across both bases, thus determining two elements on each

cone. These four elements, which lie in the same cutting plane, intersect as shown, and thus determine four points on the curve of intersection. The four elements are shown in both views. Additional planes are drawn at random, so that their intersections with the base plane cut across both bases and pass through the point N. The point N is the controlling point for the entire solution. The first planes used should always contain the extreme elements on both cones for both views, in order to locate the exact points at which the curve comes tangent to them.

This method may also be used in finding the line of intersection of two pyramids, or a pyramid and a cone.

8.7. Two Cones with Their Bases in Different and Nonparallel Planes.

First Method.

The first method of Section 7.7 will apply just the same, even though the bases are not in the same plane.

Second Method.

ANALYSIS.

A cutting plane is used which contains the line connecting the vertices of the two cones. The intersection of this plane with each of the two base planes will have to contain the point at which the line of the vertices pierces these planes. These two lines of intersection with the base planes may then be drawn in at random, one through each piercing point, provided they meet on the line of intersection of the two base planes and also provided that each line cuts across a base of a cone. The points at which these lines of intersection intersect the bases will determine two straight-line elements on each cone. These four elements will intersect to give four points on the intersection of the cones. Additional planes will give more points, but no two of these planes are parallel, since each must contain the line of the vertices.

Explanation (see Fig. 704).

The two cones are in the positions shown with their vertices at A and B. In the front elevation, the line of the vertices, AB, is seen to pierce the plane of the base of the "A" cone at N and of the B cone at M. The point M is located in the plan and the point N is located in the side elevation. These two points are

now the controlling points for the solution. In the plan, the intersection of the first cutting plane is drawn at random from M across the base of the B cone. This intersection meets the line of intersection of the two base planes at the point H. H is projected to the side elevation, and the intersection of this cutting plane with the plane of the base of the A cone must be the line NH. In the plan the two elements of the B cone are determined and in the side elevation the two elements on the A cone are determined. All four elements lie in the first cutting plane and they are all located in every view. The four points where they intersect are apparent. These points should check between views by projection and by measurement.

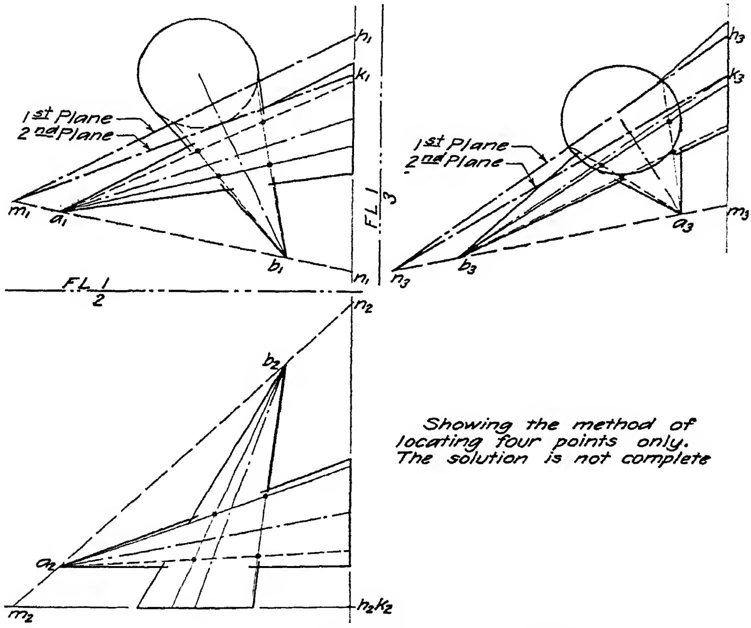


FIG. 704.—Two cones. Bases in different and nonparallel planes.

The solution is not complete in Fig. 704. A second cutting plane is shown; it intersects the two base planes in the lines MK and NK. Any other planes may be drawn in the same manner.

The solution by this second method will apply for all problems involving the intersection of two cones, two pyramids, or a pyramid with a cone.

9.7. Practice Problems.

See Chapter VIII, Group 61.

10.7. Locus of a Line. Two Right Cones of Revolution with the Same Vertex and with Elements the Same Length.

ANALYSIS.

Problems involving the intersection of two right cones of revolution arise when the position of a line in space is to be determined so as to satisfy two given angular conditions. Such problems are analyzed by thinking of the locus of the line for each condition. For example, if a line passing through some fixed point is to have a definite slope, it must lie on a cone of revolution whose vertex is the fixed point and whose elements all have the given slope. In other words, that cone is the locus of all lines, regardless of their length, that have the given slope and contain the given point. If the required line is to make a fixed angle with a given line or plane, another cone is drawn to show the locus of all possible positions of the line to satisfy this condition. Both cones *must have the same vertex and the same length elements* because they are both generated by the same line. The line of intersection of these two cones lies on both cones and therefore satisfies both given conditions.

Explanation (see Fig. 705).

A line containing the fixed point X is to be located so it makes an angle of 45° with the given plane HKM and so it has a true slope of 45°. The auxiliary elevation view 3 is drawn to show the plane as an edge and the point X is located here. Choosing any length element, such as x_3r_3, the two cones are drawn. Every element on cone A makes 45° with the plane and every element on cone B slopes 45°. In view 3 the two bases are both edges and, since they are circles, they intersect at two points, Y and Z. To locate these two points in the plan, the base of cone B is drawn and they must project on to this circle. Or the base of cone A is drawn in view 4, where Y and Z are projected to the circle and then located in the plan by measurement. The elements XY and XZ lie on both cones and are two possible positions of a line to satisfy the given conditions. The cones may be drawn any size but the elements of the two cones must always be the same length.

This problem is just one typical example of a large variety of problems which may be analyzed by the method of locus of a line and solved by the intersection of two surfaces. The given conditions fix the kind of surfaces the line must lie on so their line of intersection will satisfy these conditions. It is easily possible to have angular specifications given which would be impossible for any line in space to fulfil. In such a case the impossible situation

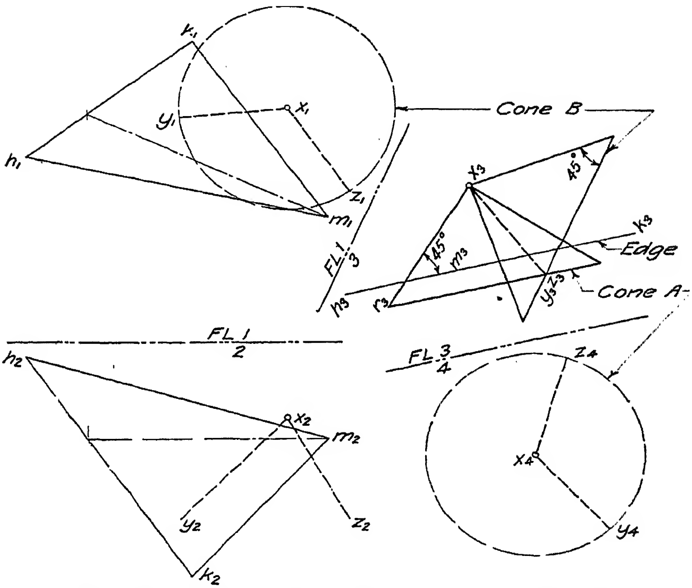


FIG. 705.—Locus of a line. Two right cones of revolution.

is evident as soon as the two cones are drawn because they lie wholly outside each other and will never intersect.

11.7. Practice Problems.

See Chapter VIII, Group 62.

12.7. Cone and Cylinder.

First Method.

By the piercing-point method and using two views only, the points at which any straight-line element on either surface pierces the other surface may easily be determined.

Second Method.

A new view may be drawn in which the cylinder appears as an edge. In this view the point at which any straight-line element on the cone intersects the cylinder will be apparent.

Third Method.

A line may be drawn through the vertex of the cone and parallel to the cylinder. A series of planes are taken so that they contain this line and cut both surfaces. These planes will cut straight-line elements on both surfaces, which will have to intersect. This solution is performed more easily in the view in which the line through the vertex appears as a point, in which case it is merely an application of the second method.

This third method will also apply for all problems involving the intersection of a cone with a prism, a pyramid with a prism, and a pyramid with a cylinder.

13.7. Practice Problems.

See Chapter VIII, Group 61.

14.7. Sphere Method.

There is a very special method for solving intersections of surfaces known as the sphere method, with which every draftsman should be familiar. It is a one-view method. Where it can be used it furnishes a very easy solution, but its use is somewhat limited. The following three conditions must be satisfied before this method may be used:

1. The intersecting surfaces must both be surfaces of revolution.
2. Their axes must intersect.
3. Their axes must both show in their true length in the view in which the solution is made.

These three conditions should always be checked before use of this method is attempted.

ANALYSIS.

If the three conditions given have been satisfied, the intersection of any surface of revolution with a sphere is a circle which appears as an edge in the view in which the solution is made. Figure 706 (*a*) and (*b*) show a cylinder and a cone, respectively,

intersecting a sphere, and the intersections, in each case, are seen to be straight lines.

At the point of intersection of the axes of the two intersecting surfaces a sphere is drawn, having any diameter. It intersects both of the given surfaces in circles which appear as straight lines. Both of these circles also lie on the surface of the sphere. If they intersect on the sphere, that point of intersection also lies on both of the given surfaces and is therefore a point on their line of

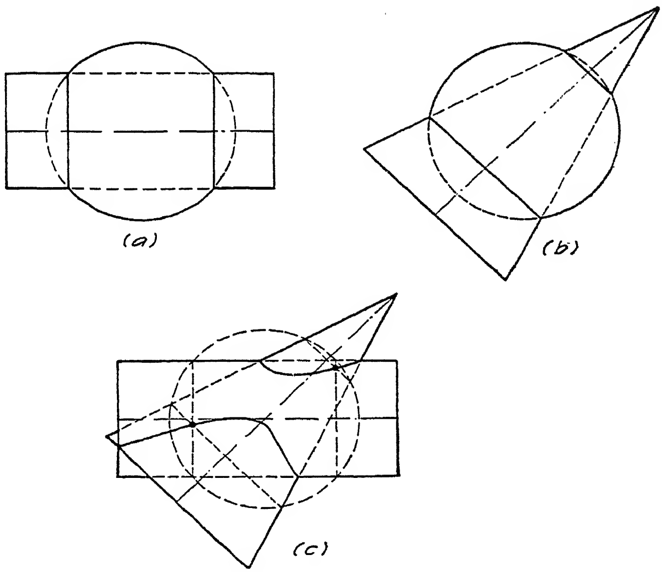


FIG. 706. – Sphere method.

intersection. A series of different-sized spheres is drawn until the curve of intersection is entirely established. The solution is given in Fig. 706 (*c*), where only one sphere is shown.

This method may be used with any combination of two surfaces of revolution provided the three conditions are satisfied. Since many actual metal pieces are surfaces of revolution with intersecting axes, this method has considerable practical value.

15.7. General Procedure.

Each combination of two intersecting surfaces presents a situation a little different, and requires careful analysis before the

solution is attempted. There is no one method by which all intersection problems may be solved. However, there is a rather general procedure of analysis which may be followed in order to arrive at the best method of solution.

The first step in this analytical procedure is to have clearly in mind the nature of each surface, the way it is generated, and the different kinds of elements which may lie upon it. If a surface is double-ruled, clear determination should be made as to which way both sets of straight-line elements lie.

The second step is to think of cutting across both the given surfaces with a plane which will contain some element on each surface. It will have to be decided which way this plane must be passed in order to cut out the simplest elements on both surfaces. Straight-line elements should always be used if possible. Circular elements are the next easiest to handle. If elements on the two given surfaces lie in this same cutting plane, they will intersect (in general) at one, two, or four points, which will be points on the line of intersection of the two surfaces.

The third step in the analysis is to determine which way other planes must be passed in order to locate sufficient points to determine the entire line of intersection. Sometimes the cutting planes are all parallel, and sometimes they are intersecting, according to the nature of the given surfaces. A sufficient number of additional cutting planes should be passed to determine completely and accurately the desired line of intersection.

Some of the more unusual intersections are now listed with a brief statement of the method recommended for the solution of each one. Each method is selected by analysis closely following the general procedure explained in this section.

1. *Two spheres.* Use a series of planes, preferably horizontal or vertical, any one of which cuts a circle from each sphere. Solve in the view where these circles appear as circles.
2. *Prism and sphere.* Use a series of planes parallel to the prism. Vertical planes give the easiest solution.
3. *Pyramid and sphere.* Use a series of planes containing the vertex of the pyramid. Vertical planes are preferable.
4. *Cylinder and sphere.* Same as for prism and sphere.
5. *Cone and sphere.* Same as for pyramid and sphere.
6. *Prism and hyperbolic paraboloid.* Use a series of planes containing straight-line elements on the warped surface and cutting straight lines on the prism surfaces.

7. *Pyramid and hyperbolic paraboloid.* Same as for prism and hyperbolic paraboloid.
8. *Cylinder and hyperbolic paraboloid.* Use a series of planes containing straight-line elements on the warped surface. These planes will cut ellipses, circles, or straight lines on the cylinder, depending upon its position relative to the warped surface. Or a series of planes could be used cutting straight-line elements on the cylinder and curved lines on the warped surface.
9. *Cone and hyperbolic paraboloid.* Use a series of planes containing the vertex of the cone and cutting curved lines on the warped surface.
10. *Torus and cylinder.* Use a series of planes perpendicular to the axis of the torus. These planes will cut circles on the torus and ellipses, circles, or straight lines on the cylinder.
11. *Torus and cone.* Use a series of planes perpendicular to the axis of the torus and cutting a series of ellipses or circles on the cone.

16.7. Miscellaneous Practice Problems.

See Chapter VIII, Group 61.

CHAPTER VIII

PRACTICE PROBLEMS

The problems contained in this chapter are given for the purpose of furnishing practice in the use of each principle which has been explained. As soon as a principle has been studied, it should be tried out by solving some of the various problems listed under that group. These problems are all given without any data, but they should be laid out approximately in the proportion and in the position as given. In a few problems the proportion between the width and height of a solid object will be suggested.

These problems are designed to test out a student's understanding of a theoretical method and his ability to apply that method in obtaining a solution in a large variety of situations.

For convenience each problem has been given a classification number, such as 8-24-7. This notation means that this problem is number 7 in Group 24 in Chapter VIII.

Group 1. Three or More Ordinary Views of an Object

In each problem draw the specified views of the object and show all hidden lines in all views. The rule in shop drawing is to omit dash lines unless they make the drawing clearer. Since the chief purpose of these problems is to give practice, the locating of the dash lines will give just that much more practice. Also show and label all the folding lines.

8-1-1. Draw the plan, the front elevation, and the right side elevation of the object in Fig. 801(1).

8-1-2. Draw the plan, the front elevation, and both side elevations of the object in Fig. 801(2).

8-1-3. Draw the plan, the front elevation, and both side elevations of the object in Fig. 801(3).

8-1-4. Draw the two given views and one side elevation of the object in Fig. 801(4).

8-1-5. Draw the two given views and both side elevations of the object in Fig. 801(5).

8-1-6. Draw the two given views and both side elevations of the object in Fig. 801(6).

8-1-7. Draw the two given views and the right side elevation of the object in Fig. 801(7).

8-1-8. Draw the two given views and the left side elevation of the object in Fig. 801(8).

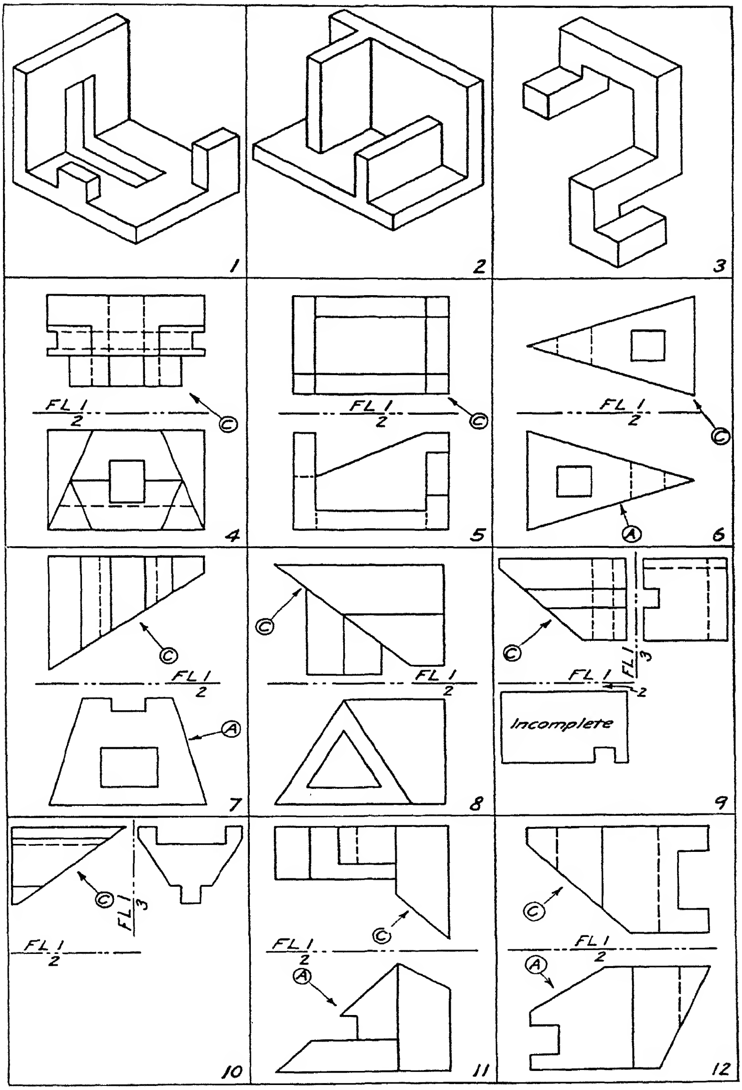

1
2
3
FL 1/2
C
4
FL 1/2
C
5
FL 1/2
C
A
6
C
FL 1/2
A
7
C
FL 1/2
8
C
FL 1/3
FL 1/2
Incomplete
9
C
FL 1/3
FL 1/2
10
FL 1/2
C
A
11
C
FL 1/2
A
12

FIG. 801.

8-1-9. Draw the plan and both side elevations and complete the front elevation of the object in Fig. 801(9).

8-1-10. Draw the plan, the right side elevation, and the front elevation of the object in Fig. 801(10).

8-1-11. Draw the two given views and the left side elevation of the object in Fig. 801(11).

8-1-12. Draw the given views and the left side elevation of the object in Fig. 801(12).

Group 2. Auxiliary Elevation Views of an Object

In each problem draw the two given views and the auxiliary elevation looking in the direction indicated by the arrow marked *C*. Show the entire object, including all dash lines, in every view.

8-2-1. Object in Fig. 802 (1).
8-2-2. Object in Fig. 801 (4).
8-2-3. Object in Fig. 801 (5).
8-2-4. Object in Fig. 801 (6).
8-2-5. Object in Fig. 801 (7).
8-2-6. Object in Fig. 801 (8).
8-2-7. Object in Fig. 801 (9) (omit the front elevation).
8-2-8. Object in Fig. 801(10) (omit the front elevation).
8-2-9. Object in Fig. 801(11).
8-2-10. Object in Fig. 801(12).

Group 3. Inclined Views Taken from the Front Elevation

In each problem draw the two given views and an inclined view looking in the direction indicated by the arrow marked *A*. Show the entire object, including all dash lines, in every view.

8-3-1. Object in Fig. 802 (1).
8-3-2. Object in Fig. 802 (2).
8-3-3. Object in Fig. 802 (3).
8-3-4. Object in Fig. 802 (4). Show only the lower part of the object as cut by the plane *AA*.
8-3-5. Object in Fig. 802 (5).
8-3-6. Object in Fig. 802 (6).
8-3-7. Object in Fig. 801 (6).
8-3-8. Object in Fig. 801 (7).
8-3-9. Object in Fig. 801(11).
8-3-10. Object in Fig. 801(12).

Group 4. Inclined Views Taken from Auxiliary Elevations or Other Inclined Views

In each problem draw the given views and obtain a view which shows the designated face in its true size. Show the entire object, including all the dash lines, in every view.

8-4-1. Face *ABC*, Fig. 802 (7).
8-4-2. Face *ABCD*, Fig. 802 (8).
8-4-3. Face *ABC*, Fig. 802 (9).

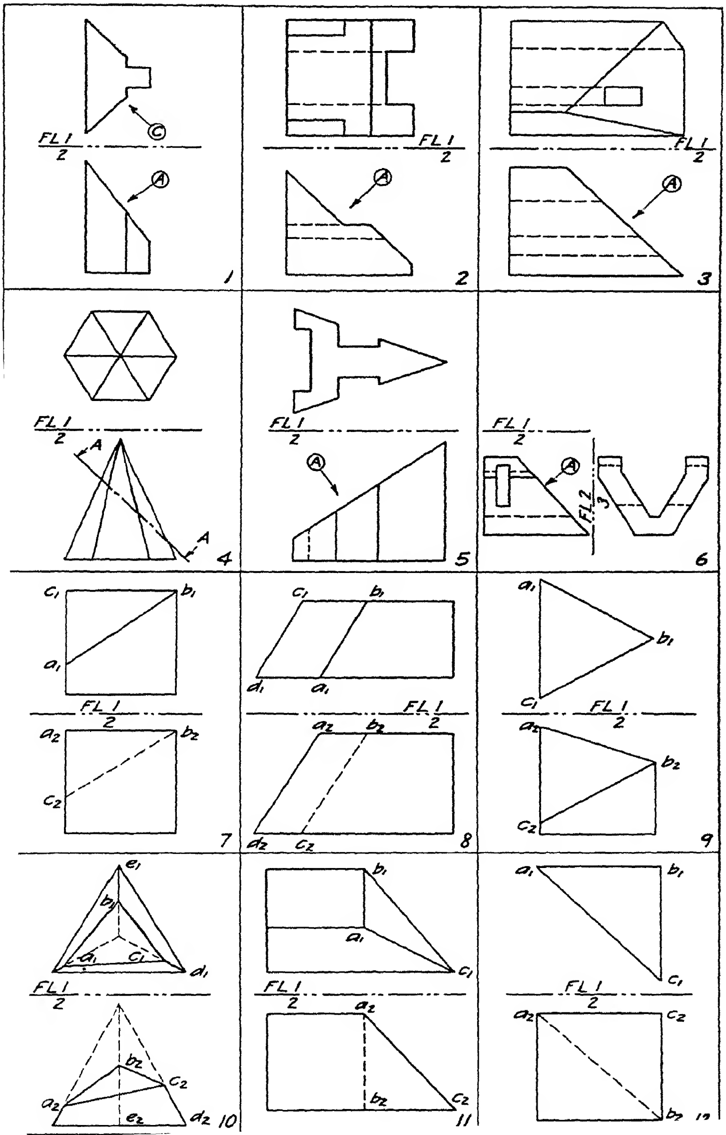


Fig. 802.

8-4-4. Face ABC, Fig. 802(10).
8-4-5. Face $BCDE$, Fig. 802(10).
8-4-6. Face ABC, Fig. 802(11).
8-4-7. Face ABC, Fig. 802(12).

Group 5. True Length of a Line

In each problem find the true length of the specified line in an elevation view. Check this length by a true-length view taken from the front elevation.

8-5-1. Line MN, Fig. 803 (4).
8-5-2. Line MN, Fig. 803 (5).
8-5-3. Line MN, Fig. 803 (6).
8-5-4. Line MN, Fig. 803 (7).
8-5-5. Line MN, Fig. 803 (8).
8-5-6. Line MN, Fig. 803 (9).
8-5-7. Line AB, Fig. 804(10).
8-5-8. Line AB, Fig. 804(12).

Group 6. True Slope of a Line

In each problem find the true slope of the specified line. Mark the slope angle in degrees and also give the per cent grade.

8-6-1. Line MN, Fig. 803 (4).
8-6-2. Line MN, Fig. 803 (5).
8-6-3. Line MN, Fig. 803 (6).
8-6-4. Line MN, Fig. 803 (7).
8-6-5. Line MN, Fig. 803 (8).
8-6-6. Line MN, Fig. 803 (9).
8-6-7. Line AB, Fig. 804(10).
8-6-8. Line CD, Fig. 804(10).

Group 7. View Showing a Line as a Point

In each problem draw a view of the specified line in which it appears as a point.

8-7-1. Line MN, Fig. 803(2).
8-7-2. Line MN, Fig. 803(3).
8-7-3. Line MN, Fig. 803(4).
8-7-4. Line MN, Fig. 803(5).
8-7-5. Line MN, Fig. 803(6).
8-7-6. Line MN, Fig. 803(7).
8-7-7. Line MN, Fig. 803(8).
8-7-8. Line MN, Fig. 803(9).

Group 8. Edge View of a Plane. General Method

In each problem draw a view of the specified plane in which it appears as an edge and the specified line on the plane appears as a point.

8-8-1. Plane ABC, line BC, Fig. 803(10).
8-8-2. Plane ABC, line AB, Fig. 803(11).
8-8-3. Plane ABC, line BC, Fig. 803(12).
8-8-4. Plane ABC, line AB, Fig. 804 (1).

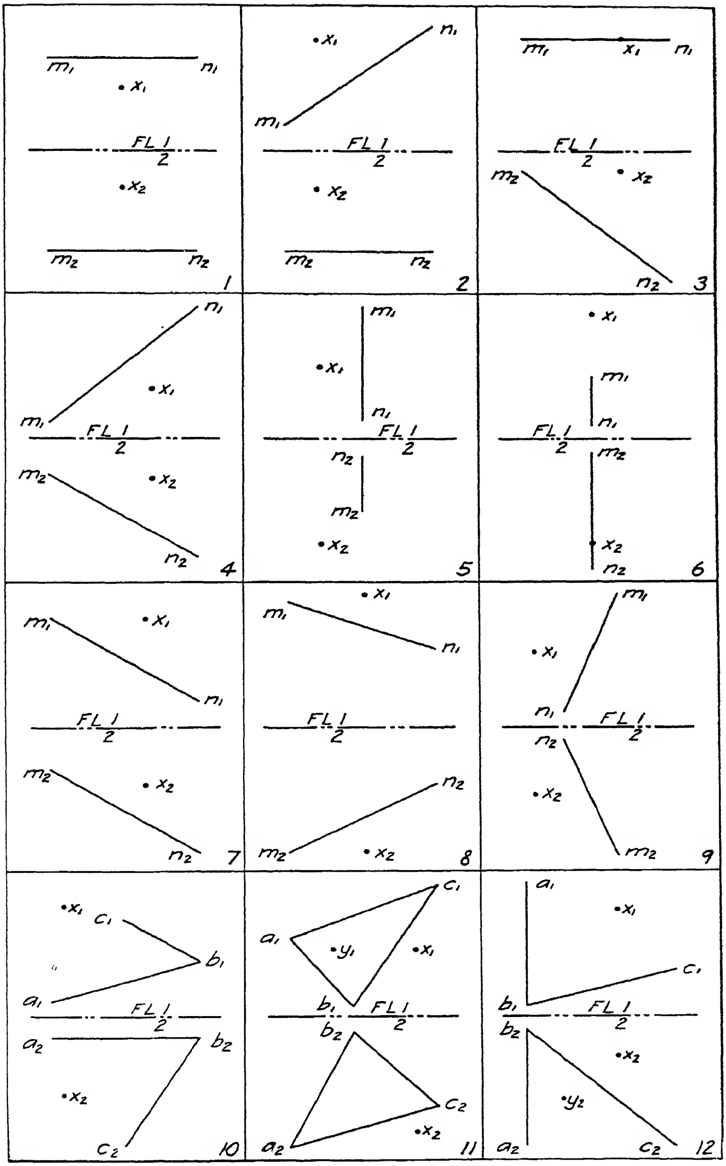

m1 n1 x1 FL 1/2 x2 m2 n2 1
x1 n1 m1 FL 1/2 x2 m2 n2 2
m1 x1 n1 FL 1/2 m2 x2 n2 3
n1 x1 m1 FL 1/2 m2 x2 n2 4
m1 x1 n1 FL 1/2 n2 m2 x2 5
x1 m1 n1 FL 1/2 m2 x2 n2 6
m1 x1 n1 FL 1/2 m2 x2 n2 7
x1 m1 n1 FL 1/2 n2 m2 x2 8
m1 x1 n1 FL 1/2 n2 x2 m2 9
x1 c1 b1 a1 FL 1/2 a2 b2 x2 c2 10
c1 a1 y1 x1 b1 FL 1/2 b2 c2 x2 a2 11
a1 x1 c1 b1 FL 1/2 b2 x2 y2 a2 c2 12

Fig. 803.

8-8-5. Plane $ABCD$, line AB, Fig. 804 (2).
8-8-6. Plane ABC, line AB, Fig. 804 (5).

Group 9. Edge View of a Plane. Special Methods

In each problem show the specified plane as an edge by drawing only one additional view.

8-9-1. Plane ABC, Fig. 803(10). By the method of Case 1.
8-9-2. Plane ABC, Fig. 804 (1). By the method of Case 1.
8-9-3. Plane ABC, Fig. 804 (5). By the method of Case 1.
8-9-4. Plane ABC, Fig. 803(11). By the method of Case 2.
8-9-5. Plane ABC, Fig. 803(12). By the method of Case 2.
8-9-6. Plane $ABCD$, Fig. 804 (2). By the method of Case 2.

Group 10. True Slope of a Plane

In each problem find the true slope of the designated plane.

8-10-1. Plane ABC, Fig. 803(10).
8-10-2. Plane ABC, Fig. 803(11).
8-10-3. Plane ABC, Fig. 803(12).
8-10-4. Plane ABC, Fig. 804 (1).
8-10-5. Plane ABC, Fig. 804 (3).
8-10-6. Plane ABC, Fig. 804 (4).

Group 11. True Size of a Plane, Using Only Two Extra Views

In each problem draw the view looking at right angles to the plane, by two methods.

I. By taking an edge view directly from the plan.
II. By taking an edge view directly from the front elevation.

The size of the plane is assumed to mean that part of the plane within the limits of the lines which are used to fix it in space.

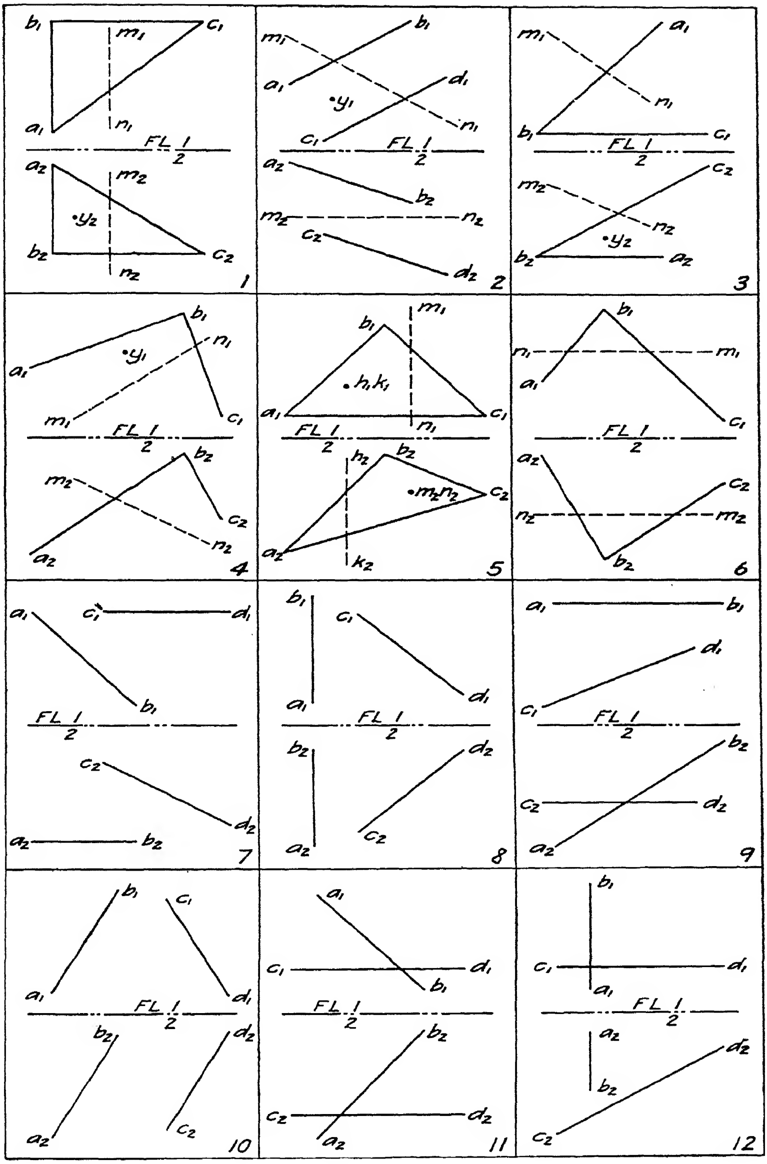
8-11-1. Plane ABC, Fig. 803(10).
8-11-2. Plane ABC, Fig. 803(11).
8-11-3. Plane ABC, Fig. 803(12).
8-11-4. Plane ABC, Fig. 804 (1).
8-11-5. Plane $ABCD$, Fig. 804 (2).
8-11-6. Plane ABC, Fig. 804 (3).
8-11-7. Plane ABC, Fig. 804 (4).
8-11-8. Plane ABC, Fig. 804 (5).
8-11-9. Plane ABC, Fig. 804 (6).

Group 12. Various Views of Any Point in a Plane

In each problem the plane is given, and one view of a point which is on that plane. Show the point in the other view by two different methods.

I. By using two views only.
II. By the edge-view method.

8-12-1. Point Y on the plane ABC, Fig. 803(11).
8-12-2. Point Y on the plane ABC, Fig. 803(12).
8-12-3. Point Y on the plane ABC, Fig. 804 (1).
8-12-4. Point Y on the plane $ABCD$, Fig. 804 (2).
8-12-5. Point Y on the plane ABC, Fig. 804 (3).
8-12-6. Point Y on the plane ABC, Fig. 804 (4).



b1 c1 m1 a1 n1 FL 1/2 a2 m2 y2 b2 c2 n2 1
m1 b1 a1 d1 y1 c1 n1 FL 1/2 a2 b2 m2 n2 c2 d2 2
m1 a1 n1 b1 c1 FL 1/2 c2 m2 n2 y2 b2 a2 3
b1 n1 a1 y1 c1 m1 FL 1/2 b2 m2 c2 n2 a2 4
m1 b1 h1k1 a1 c1 n1 FL 1/2 h2 b2 m2n2 c2 a2 k2 5
b1 n1 m1 a1 c1 FL 1/2 a2 c2 n2 m2 b2 6
a1 c1 d1 b1 FL 1/2 c2 d2 a2 b2 7
b1 c1 d1 a1 FL 1/2 b2 d2 c2 a2 8
a1 b1 d1 c1 FL 1/2 b2 c2 d2 a2 9
b1 c1 a1 d1 FL 1/2 b2 d2 a2 c2 10
a1 c1 d1 b1 FL 1/2 b2 c2 d2 a2 11
b1 c1 d1 a1 FL 1/2 a2 d2 b2 c2 12

Fig. 804.

Group 13. Line Having a Given Bearing and a Given Slope

In each problem assume the true length of the specified line to be about 3 in. and show the line in the plan and front elevation views.

8-13-1. Line AB bearing N 45° E from A and falling 30 degrees.

8-13-2. Line AB bearing S 15° W from A and rising on a 100-per cent grade.

8-13-3. Line AB bearing N 30° W from A and having a slope $= -0.5$.

8-13-4. Line AB bearing S 75° E from A and rising on a 45-per cent grade.

8-13-5. Line AB bearing N 75° W from A and rising 60 degrees.

8-13-6. Line AB bearing S 30° E from A and having a slope $= +7/10$.

Group 14. Distance from a Point to a Line

In each problem find the true distance from the given point to the given line by two methods.

I. By the line method.

II. By the plane method.

8-14-1. Point X to line MN, Fig. 803 (4).

8-14-2. Point X to line MN, Fig. 803 (5).

8-14-3. Point X to line MN, Fig. 803 (6).

8-14-4. Point X to line MN, Fig. 803 (7).

8-14-5. Point X to line MN, Fig. 803 (8).

8-14-6. Point X to line MN, Fig. 803 (9).

8-14-7. Point X to line AC, Fig. 803(11).

8-14-8. Point X to line MN, Fig. 803 (2).

Group 15. Plane Parallel to One Line and Containing Another Line

In each problem draw a plane which contains the line AB and is parallel to the line CD. As a check, draw an edge view of the plane and see whether the line CD lies parallel to the plane.

8-15-1. Lines AB and CD, Fig. 804 (7).

8-15-2. Lines AB and CD, Fig. 804 (8).

8-15-3. Lines AB and CD, Fig. 804 (9).

8-15-4. Lines AB and CD, Fig. 804(10).

8-15-5. Lines AB and CD, Fig. 804(11).

8-15-6. Lines AB and CD, Fig. 804(12).

Group 16. Shortest Distance between Two Lines. Line Method

In each problem find the location, true length, and true slope of the shortest line to connect the two given lines. Use the line method of section 12.3.

8-16-1. Lines AB and CD, Fig. 804 (7).

8-16-2. Lines AB and CD, Fig. 804 (8).

8-16-3. Lines AB and CD, Fig. 804 (9).

8-16-4. Lines AB and CD, Fig. 804(10).

8-16-5. Lines AB and CD, Fig. 804(11).

8-16-6. Lines AB and CD, Fig. 804(12).

Group 17. Shortest Distance between Two Lines. Plane Method

In each problem find the location, true length, and true slope of the shortest distance between the two given lines. Use the plane method of section 12.3

8-17-1. Lines AB and CD, Fig. 804 (7).
8-17-2. Lines AB and CD, Fig. 804 (8).
8-17-3. Lines AB and CD, Fig. 804 (9).
8-17-4. Lines AB and CD, Fig. 804(10).
8-17-5. Lines AB and CD, Fig. 804(11).
8-17-6. Lines AB and CD, Fig. 804(12).

Group 18. Shortest Level Line between Two Lines

In each problem find the location, true length, and bearing of the shortest level line to connect the two given lines.

8-18-1. Lines AB and CD, Fig. 804 (7).
8-18-2. Lines AB and CD, Fig. 804 (8).
8-18-3. Lines AB and CD, Fig. 804 (9).
8-18-4. Lines AB and CD, Fig. 804(10).
8-18-5. Lines AB and CD, Fig. 804(11).
8-18-6. Lines AB and CD, Fig. 804(12).

Group 19. Line Piercing a Plane

In each problem show in both the given views where the given line pierces the given plane. Solve using two views only, and check by the edge-view method.

8-19-1. Line MN, plane ABC, Fig. 804(1). Extra view allowed.
8-19-2. Line MN, plane $ABCD$, Fig. 804(2).
8-19-3. Line MN, plane ABC, Fig. 804(3).
8-19-4. Line MN, plane ABC, Fig. 804(4).
8-19-5. Line MN, plane ABC, Fig. 804(5).
8-19-6. Line HK, plane ABC, Fig. 804(5).
8-19-7. Line MN, plane ABC, Fig. 804(6).

Group 20. Intersection of Two Planes

In each problem find the line of intersection of the two specified planes, which should be placed beside each other and close together. Locate points on the line of intersection by all three methods.

I. Piercing-point method.
II. Auxiliary vertical-cutting-plane method.
III. Auxiliary horizontal-cutting-plane method.

8-20-1. Plane ABC in Fig. 803(11) with plane ABC in Fig. 803(12).
8-20-2. Plane ABC in Fig. 804 (1) with plane $ABCD$ in Fig. 804 (2).
8-20-3. Plane ABC in Fig. 804 (1) with plane ABC in Fig. 804 (3).
8-20-4. Plane ABC in Fig. 804 (1) with plane ABC in Fig. 804 (4).
8-20-5. Plane ABC in Fig. 804 (3) with plane ABC in Fig. 804 (6).
8-20-6. Plane ABC in Fig. 804 (5) with plane $ABCD$ in Fig. 804 (2).

Group 21. Dihedral Angle

In each problem draw the two planes as they are specified, by connecting the given points. Find the true size of the dihedral angle between the two specified planes.

8-21-1. Planes *ABX* and *ABC*, Fig. 803(10).
8-21-2. Planes *ABX* and *ABC*, Fig. 803(11).
8-21-3. Planes *ABC* and *BCX*, Fig. 803(12).
8-21-4. Planes *ACX* and *ABC*, Fig. 803(11).
8-21-5. Planes *ABC* and *BCD*, Fig. 804 (7).
8-21-6. Planes *ABC* and *ACD*, Fig. 804 (8).
8-21-7. Planes *ABD* and *ADC*, Fig. 804 (9).
8-21-8. Planes *ABC* and *ACD*, Fig. 804(10).
8-21-9. Planes *ABD* and *BDC*, Fig. 804(11).
8-21-10. Planes *ABD* and *ADC*, Fig. 804(12).

Group 22. Distance from a Point to a Plane

In each problem, using the two given views only, show the perpendicular from the given point to the given plane and show where this perpendicular pierces the plane. Check by drawing an edge view of the plane and measure the true length in this view.

8-22-1. Point *X* to plane *ABC*, Fig. 803(10).
8-22-2. Point *X* to plane *ABC*, Fig. 803(11).
8-22-3. Point *X* to plane *ABC*, Fig. 803(12).
8-22-4. Point *M* to plane *ABCD*, Fig. 804 (2).
8-22-5. Point *M* to plane *ABC*, Fig. 804 (3).
8-22-6. Point *M* to plane *ABC*, Fig. 804 (4).
8-22-7. Point *H* to plane *ABC*, Fig. 804 (5).
8-22-8. Point *N* to plane *ABC*, Fig. 804 (5).
8-22-9. Point *M* to plane *ABC*, Fig. 804 (6).
8-22-10. Point *C* to plane *ABD*, Fig. 804 (8).

Group 23. Project a Line on to a Plane

In each problem show, in both the given views, the projection of the specified line on to the specified plane.

8-23-1. Line *MN* on plane *ABCD*, Fig. 804(2).
8-23-2. Line *MN* on plane *ABC*, Fig. 804(3).
8-23-3. Line *MN* on plane *ABC*, Fig. 804(4).
8-23-4. Line *MN* on plane *ABC*, Fig. 804(5).
8-23-5. Line *HK* on plane *ABC*, Fig. 804(5).
8-23-6. Line *MN* on plane *ABC*, Fig. 804(6).

Group 24. Angle between a Line and a Plane

In each problem find the true size of the angle between the specified line and the specified plane.

8-24-1. Line *MN* and plane *ABC*, Fig. 804 (1).
8-24-2. Line *MN* and plane *ABCD*, Fig. 804 (2).
8-24-3. Line *MN* and plane *ABC*, Fig. 804 (3).
8-24-4. Line *MN* and plane *ABC*, Fig. 804 (4).

8-24-5. Line MN and plane ABC, Fig. 804 (5).
8-24-6. Line HK and plane ABC, Fig. 804 (5).
8-24-7. Line MN and plane ABC, Fig. 804 (6).
8-24-8. Line CD and plane ABC, Fig. 804 (7).
8-24-9. Line CD and plane ABC, Fig. 804 (8).
8-24-10. Line AB and plane ACD, Fig. 804 (9).
8-24-11. Line CD and plane ABC, Fig. 804(10).
8-24-12. Line CD and plane ABC, Fig. 804(11).
8-24-13. Line AB and plane ACD, Fig. 804(12).

Group 25. To Draw a Plane Figure on Any Oblique Plane

In each problem show in the given views the specified figure lying in the specified plane. Assume the figure to be about half the size of the plane.

8-25-1. In plane ABC, Fig. 803(10), show a square with two sides level.

8-25-2. In plane ABC, Fig. 803(12), show an equilateral triangle with one side level.

8-25-3. In plane $ABCD$, Fig. 804(2), show a hexagon having two sides parallel to the two given lines on the plane.

8-25-4. In plane ABC, Fig. 804(4), show a square having two sides that are frontal lines.

8-25-5. In plane ABC, Fig. 804(6), show an equilateral triangle with one side on the line BC.

8-25-6. In plane ABC, Fig. 804(3), show a hexagon having one side parallel to the line AB.

Group 26. To Draw a Circle on Any Oblique Plane

In each problem show in the given views a circle lying in the given plane with its center located as specified.

8-26-1. Plane ABC, Fig. 804(3), center at A.

8-26-2. Plane ABC, Fig. 804(5), inscribe the circle in the given triangular plane.

8-26-3. Plane ABC, Fig. 803(11), center at A.

8-26-4. Plane $ABCD$, Fig. 804(2), circle tangent to the two given lines, AB and CD.

Group 27. To Show a Solid Object Resting on an Oblique Plane

In each problem show the specified object on the plane in both the given views. "Standing" on the plane means that the axis is at right angles to the plane. "Lying" on the plane means that one face, not the base, coincides with the plane. The term "units" is used just to give some proportion between the altitude and the size of the base.

8-27-1. A right square prism standing on the plane ABC, Fig. 803(11). Two base edges are frontal lines. Altitude 1 unit, base 2 units.

8-27-2. A right triangular prism standing on the plane $ABCD$, Fig. 804(2). One edge of the base is level. Base edge 1 unit, altitude 2 units.

8-27-3. A right square prism lying on the plane ABC, Fig. 804(3). Axis of prism parallel to AB. Base edge 1 unit, altitude 2 units.

8-27-4. A right square pyramid lying on the plane ABC, Fig. 804(4). One base edge is on the line BC. Base edge 1 unit, altitude 3 units.

8-27-5. A right triangular pyramid standing on the plane ABC, Fig. 804(3). One base edge is level. Base edge 1 unit, altitude 2 units.

8-27-6. A right triangular pyramid lying on the plane ABC, Fig. 804(5). One base edge is a frontal line. Base edge 1 unit, altitude 2 units.

8-27-7. A thin circular disk standing on the plane ABC, Fig. 804(6).

8-27-8. A thin circular disk lying on the plane ABC, Fig. 804(5). Axis parallel to the line AB.

Group 28. Revolution of a Point

In each problem revolve the given point about the given line as an axis and through the specified angle. Show the new position of the point in both the given views and call the new position X'. If there are two possible solutions, show only one of them

8-28-1. Fig. 803(1). Revolve X about MN through a 45-degree angle.

8-28-2. Fig. 803(2). Revolve X about MN through a 90-degree angle.

8-28-3. Fig. 803(3). Revolve X about MN through a 45-degree angle.

8-28-4. Fig. 803(4). Revolve X about MN through a 30-degree angle.

8-28-5. Fig. 803(5). Revolve X about MN through a 60-degree angle.

8-28-6. Fig. 803(6). Revolve X about MN through a 90-degree angle.

8-28-7. Fig. 803(7). Revolve X about MN through a 45-degree angle.

8-28-8. Fig. 803(8). Revolve X about MN until it lies in the same vertical plane as MN.

8-28-9. Fig. 803(9). Revolve X about MN until it appears on m_2n_2.

8-28-10. Fig. 803(5). Revolve X about MN until it lies at the same level as M.

Group 29. Revolution of a Line

In each problem revolve the given line until it shows in its true length in the plan. Check the result by revolving the line until it shows in its true length in the front elevation.

8-29-1. Line MN, Fig. 803(4).

8-29-2. Line MN, Fig. 803(5).

8-29-3. Line MN, Fig. 803(6).

8-29-4. Line MN, Fig. 803(7).

8-29-5. Line MN, Fig. 803(8).

8-29-6. Line MN, Fig. 803(9).

Group 30. Revolution of a Plane

In each problem revolve the given plane until it shows in its true size in the plan. Also revolve the plane until it shows in its true size in the front elevation, and see whether the two results check.

8-30-1. Plane ABC, Fig. 803(10).

8-30-2. Plane ABC, Fig. 803(11).

8-30-3. Plane ABC, Fig. 803(12).

8-30-4. Plane ABC, Fig. 804 (1).

8-30-5. Plane $ABCD$, Fig. 804 (2).

8-30-6. Plane ABC, Fig. 804 (3).

8-30-7. Plane ABC, Fig. 803 (4).
8-30-8. Plane ABC, Fig. 803 (5).
8-30-9. Plane ABC, Fig. 803 (6).

In each of the following problems revolve the given plane about the specified axis until it shows in its true size in some view.

8-30-10. Plane ABC, axis AB, Fig. 803(10).
8-30-11. Plane ABC, axis BC, Fig. 803(11).
8-30-12. Plane ABC, axis AB, Fig. 803(12).
8-30-13. Plane ABC, axis BC, Fig. 804 (1).
8-30-14. Plane $ABCD$, axis CD, Fig. 804 (2).
8-30-15. Plane ABC, axis AB, Fig. 804 (4).
8-30-16. Plane ABC, axis BC, Fig. 804 (6).

Group 31. Dihedral Angle by Revolution

In each problem find the true size of the dihedral angle between the two given planes by the method of revolution.

8-31-1. Planes ABC and ABX, Fig. 803(10).
8-31-2. Planes ABC and ABX, Fig. 803(11).
8-31-3. Planes ABC and ABX, Fig. 803(12).
8-31-4. Planes ABC and ABM, Fig. 804 (3).
8-31-5. Planes ABC and BCN, Fig. 804 (6).
8-31-6. Planes ABC and BCM, Fig. 804 (5).
8-31-7. Planes ABC and BCD, Fig. 804 (7).
8-31-8. Planes ABC and ABD, Fig. 804 (9).

Group 32. Angle between a Line and a Plane by Revolution

8-32-1. Line MN and plane ABC, Fig. 804(1).
8-32-2. Line MN and plane $ABCD$, Fig. 804(2).
8-32-3. Line MN and plane ABC, Fig. 804(3).
8-32-4. Line MN and plane ABC, Fig. 804(4).
8-32-5. Line MN and plane ABC, Fig. 804(5).
8-32-6. Line HK and plane ABC, Fig. 804(5).
8-32-7. Line MN and plane ABC, Fig. 804(6).

Group 33. Noncoplanar Forces

In each problem the only known force is given some value. Find the value of the unknown force in each member of the structure caused by the given load.

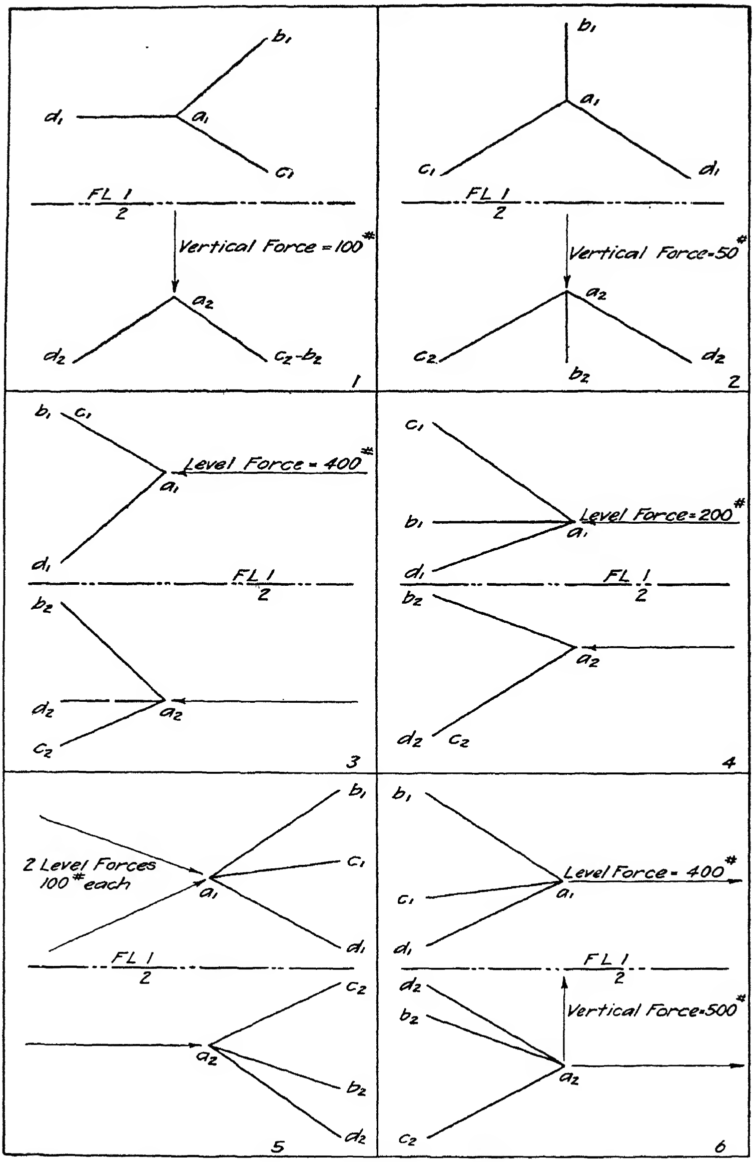
8-33-1. Three members of a tripod structure supporting a vertical load of 100 lb. Fig. 805(1).

8-33-2. Three members of a tripod structure supporting a vertical load of 50 lb. Fig. 805(2).

8-33-3. Three members of a structural frame resisting a horizontal thrust load of 400 lb. Fig. 805(3).

8-33-4. Three members of a structural frame resisting a horizontal thrust load of 200 lb. Fig. 805(4).

8-33-5. Three members of a structural frame resisting two horizontal thrust loads of 100 lb. each. Fig. 805(5).



b1
d1
a1
c1
FL 1/2
Vertical Force = 100#
a2
d2
c2-b2
1
b1
a1
c1
d1
FL 1/2
Vertical Force=50#
a2
c2
b2
d2
2
b1 c1
Level Force = 400#
a1
d1
FL 1/2
b2
d2
a2
c2
3
c1
b1
Level Force=200#
a1
d1
FL 1/2
b2
a2
d2 c2
4
b1
c1
2 Level Forces
100# each
a1
d1
FL 1/2
c2
a2
b2
d2
5
b1
Level Force = 400#
c1
a1
d1
FL 1/2
d2
b2
Vertical Force=500#
a2
c2
6

Fig. 805.

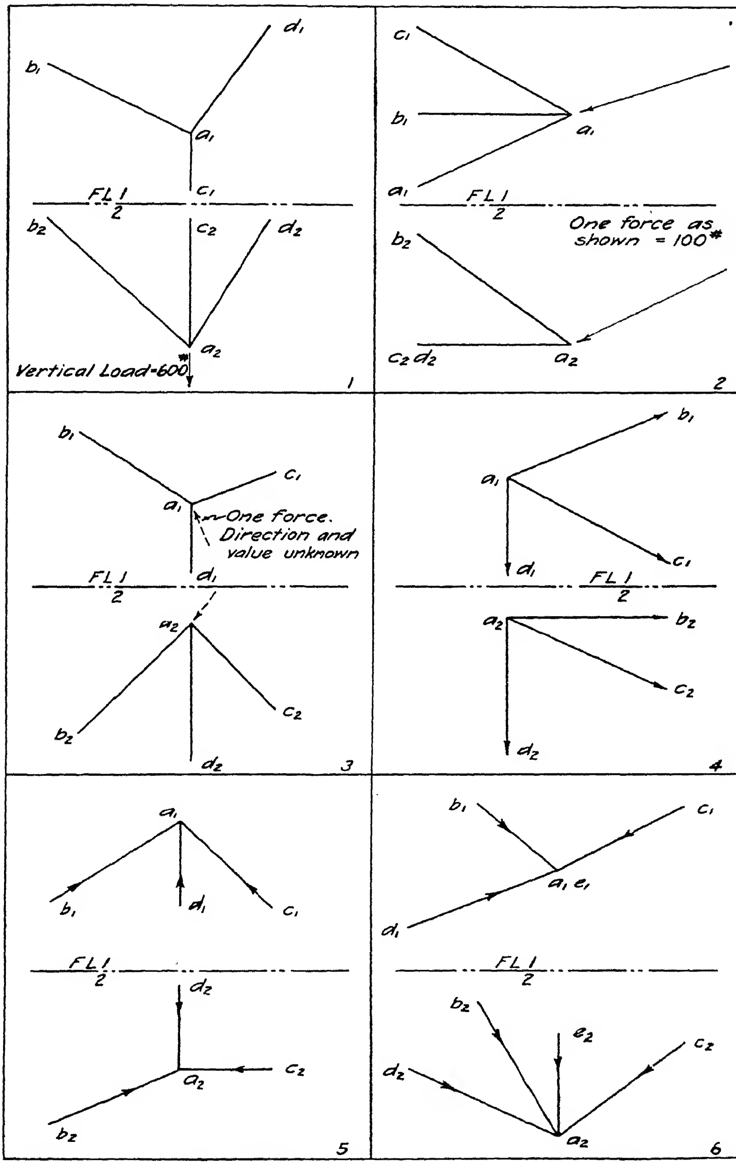

Vertical Load=600#
One force as shown = 100#
One force. Direction and value unknown
FL 1/2

FIG. 806.

8-33-6. Three members of a structural frame resisting a level pull of 400 lb. and a vertical force of 500 lb. Fig. 805(6).

8-33-7. Three members of a structural frame supporting a hanging load of 600 lb. Fig. 806(1).

8-33-8. Three members of a structural frame resisting an angular thrust load of 100 lb. Fig. 806(2).

8-33-9. Three members in compression; the force on each should be given some value. Find the direction and the value of the load which would cause the given forces in the members. Fig. 806(3).

Group 34. Resultants and Equilibrants

In each problem the vectors shown are supposed to be known. Assume a value for each, and make the required solution.

8-34-1. Three velocity vectors. Fig. 806(4). Find their resultant.

8-34-2. Three velocity vectors. Fig. 806(5). Find their resultant.

8-34-3. Four known forces. Fig. 806(6). Find their equilibrant

Group 35. Helix

8-35-1. Show two views of a right-hand cylindrical helix with a level axis. Show only one turn about the cylinder, and use 16 points to determine the curve.

8-35-2. Show two views of a left-hand cylindrical helix with a vertical axis. Show only one turn about the cylinder, and use 16 points to determine the curve.

8-35-3. Show two views of a right-hand conical helix. The cone of revolution has a vertical axis. Assume the lead to be one-third of the altitude of the cone. Show the three turns of the helix.

8-35-4. Show two views of a left-hand conical helix. The cone of revolution has a level axis. Assume the lead to be half the altitude of the cone. Show the entire helix.

Group 36. Cylinder Representation

In each problem show the plan and front elevation of a right cylinder of revolution having a specified line for an axis. The relative proportions of the cylinder are given in each problem.

8-36-1. Diameter 2 units, altitude 1 unit. Axis line MN, Fig. 803(2).

8-36-2. Diameter 1 unit, altitude 2 units. Axis line MN, Fig. 803(4).

8-36-3. Diameter 1 unit, altitude 1 unit. Axis line MN, Fig. 803(5).

8-36-4. Diameter 1 unit, altitude 2 units. Axis line MN, Fig. 803(7).

8-36-5. Diameter 3 units, altitude 1 unit. Axis line MN, Fig. 803(8).

8-36-6. Diameter 1 unit, altitude 2 units. Axis line MN, Fig. 803(9).

Group 37. Cylinder Pierced by a Line

In each problem determine whether or not the line MN pierces the given cylinder. If it pierces the cylinder, show the two piercing points in both views and indicate whether or not they are visible. Solve, using only the two given views and check by the edge-view method.

8-37-1. Fig. 807(1).

8-37-2. Fig. 807(2).

8-37-3. Fig. 807(3).
8-37-4. Fig. 807(4).
8-37-5. Fig. 807(5).
8-37-6. Fig. 807(6). Extra view allowed.

Group 38. Vertical Cylinder Cut by an Oblique Plane

In each problem draw a vertical cylinder of revolution and assume it to be cut clear across by the specified plane. Show the cut surface in the front elevation.

8-38-1. Plane *ABC*, Fig. 804(1).
8-38-2. Plane *ABCD*, Fig. 804(2).
8-38-3. Plane *ABC*, Fig. 804(3).
8-38-4. Plane *ABC*, Fig. 804(4).
8-38-5. Plane *ABC*, Fig. 804(5).
8-38-6. Plane *ABC*, Fig. 804(6).

Group 39. Development of a Vertical Cylinder

In each problem draw a vertical cylinder of revolution and assume it to be cut clear across by the specified plane. Develop either part of the cut cylinder surface.

8-39-1. Plane *ABC*, Fig. 803(10).
8-39-2. Plane *ABC*, Fig. 803(11).
8-39-3. Plane *ABC*, Fig. 803(12).
8-39-4. Plane *ABC*, Fig. 804 (1).
8-39-5. Plane *ABC*, Fig. 804 (5).
8-39-6. Plane *ABC*, Fig. 804 (6).

Group 40. Cylinder of Revolution Extending to a Level Plane

In each problem let the line *MN* be the axis of a cylinder of revolution. One end of the cylinder is cut off by a level plane containing the point *M*. Show the level end of the cylinder as it appears and show the other end with a broken line.

8-40-1. Fig. 803(3).
8-40-2. Fig. 803(4).
8-40-3. Fig. 803(5).
8-40-4. Fig. 803(7).
8-40-5. Fig. 803(8).
8-40-6. Fig. 803(9).

Group 41. Cylinder of Revolution Extending to a Frontal Plane

In each problem let the line *MN* be the axis of a cylinder of revolution. One end of the cylinder is cut off by a frontal plane containing the point *N*. Show the vertical end of the cylinder as it appears, and show the other end with a broken line.

8-41-1. Fig. 803(2).
8-41-2. Fig. 803(4).
8-41-3. Fig. 803(5).
8-41-4. Fig. 803(7).

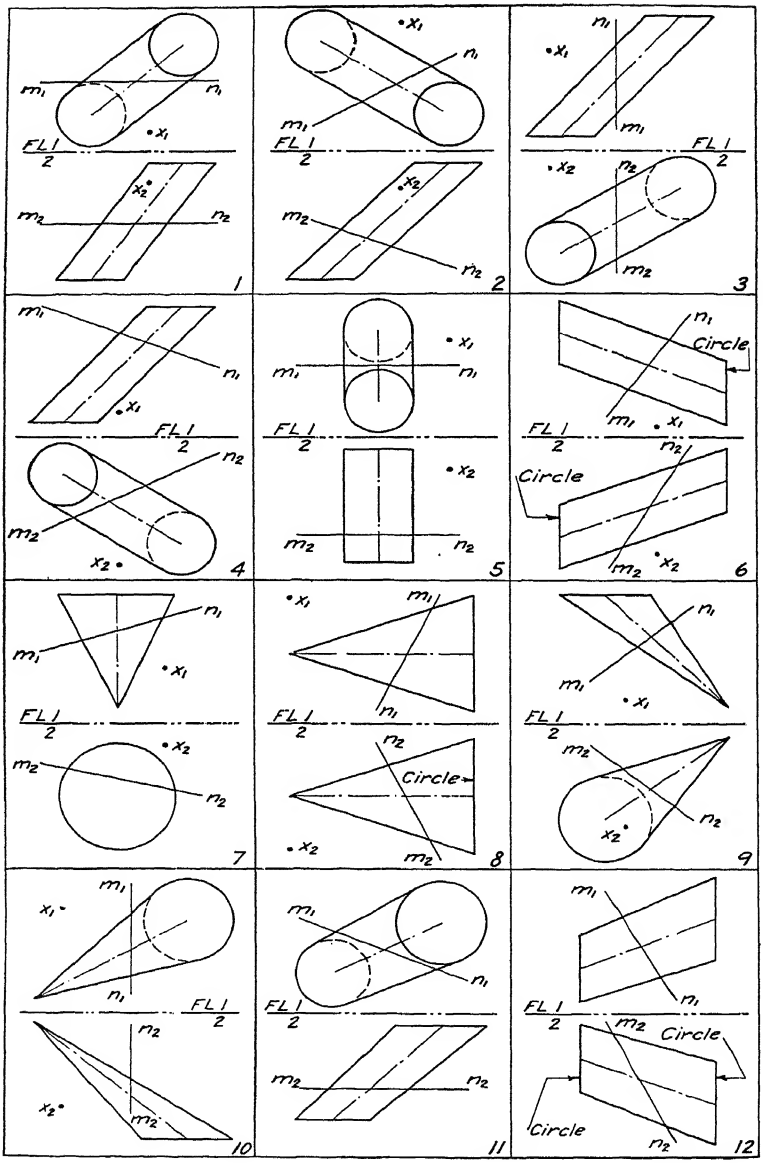

m1 n1 x1 FL 1 2 x2 m2 n2 1
x1 n1 m1 FL 1 2 x2 m2 n2 2
n1 x1 m1 FL 1 2 x2 n2 m2 3
m1 n1 x1 FL 1 2 n2 m2 x2 4
x1 m1 n1 FL 1 2 x2 m2 n2 5
n1 Circle m1 x1 FL 1 2 n2 Circle m2 x2 6
n1 m1 x1 FL 1 2 x2 m2 n2 7
x1 m1 n1 FL 1 2 n2 Circle x2 m2 8
n1 m1 x1 FL 1 2 m2 n2 x2 9
m1 x1 n1 FL 1 2 n2 x2 m2 10
m1 n1 FL 1 2 m2 n2 11
m1 n1 FL 1 2 m2 Circle Circle n2 12

Fig. 807.

8-41-5. Fig. 803(8).
8-41-6. Fig. 803(9).

Group 42. Oblique Cylinder Cut by a Vertical Plane

In each problem assume the given cylinder to be cut entirely across by a vertical plane which is not parallel to the front image plane. Show either part of the cylinder in the front elevation and develop the part shown.

8-42-1. Fig. 807(1).
8-42-2. Fig. 807(2).
8-42-3. Fig. 807(3).
8-42-4. Fig. 807(4).
8-42-5. Fig. 807(5).
8-42-6. Fig. 807(6).

Group 43. Oblique Cylinder Cut by an Oblique Plane

In each problem assume the given cylinder to be cut clear across by a sloping plane appearing as an edge in the front elevation. Show either part of the cylinder in the plan and develop the part shown.

8-43-1. Fig. 807(1).
8-43-2. Fig. 807(2).
8-43-3. Fig. 807(3).
8-43-4. Fig. 807(4).
8-43-5. Fig. 807(5).
8-43-6. Fig. 807(6).

In each problem assume the given cylinder to be cut clear across by the plane *MNX* which is unlimited in extent. Show either part of the cylinder in both the given views, and develop the part shown.

8-43-7. Fig. 807(1).
8-43-8. Fig. 807(2).
8-43-9. Fig. 807(3).
8-43-10. Fig. 807(4).
8-43-11. Fig. 807(5).
8-43-12. Fig. 807(6).

Group 44. Cylinder and Tangent Plane

In each problem show in the two given views a plane which is tangent to the given cylinder and contains the point *X*. Solve, using only two views and check by a view showing the tangent plane as an edge.

8-44-1. Fig. 807(1).
8-44-2. Fig. 807(2).
8-44-3. Fig. 807(3).
8-44-4. Fig. 807(4).
8-44-5. Fig. 807(5).
8-44-6. Fig. 807(6). Extra view allowed.

Group 45. Cone Representation

In each problem show the two views of a right cone of revolution having a specified line as an axis. The relative proportions of the cone are given for each problem. Place the vertex of the cone at the point *M*.

8-45-1. Diameter 1 unit, altitude 3 units. Axis line *MN*, Fig. 803(3).
8-45-2. Diameter 2 units, altitude 2 units. Axis line *MN*, Fig. 803(4).
8-45-3. Diameter 3 units, altitude 2 units. Axis line *MN*, Fig. 803(6).
8-45-4. Diameter 1 unit, altitude 2 units. Axis line *MN*, Fig. 803(7).
8-45-5. Diameter 1 unit, altitude 3 units. Axis line *MN*, Fig. 803(8).
8-45-6. Diameter 1 unit, altitude 2 units. Axis line *MN*, Fig. 803(9).

Group 46. Cone Pierced by a Line

In each problem determine whether or not the given line *MN* pierces the given cone. If it pierces the cone, show the two piercing points in both the given views, indicating whether they are visible. Solve, using only the two given views, and check by the edge-view method.

8-46-1. Fig. 807 (7).
8-46-2. Fig. 807 (8). Extra view allowed.
8-46-3. Fig. 807 (9).
8-46-4. Fig. 807(10).
8-46-5. Fig. 807(11). Vertex not available.
8-46-6. Fig. 807(12). Vertex not available. Extra view allowed.

Group 47. Cone of Revolution Cut by a Plane

In each problem assume the given cone of revolution to be cut across by the given plane. Show the cut in both the given views and find its true shape.

8-47-1. Cone in Fig. 807(7), cut by a vertical plane not parallel to the base or containing the vertex.

8-47-2. Cone in Fig. 807(7), cut by a plane showing as an edge in the front elevation and not containing the vertex.

8-47-3. Cone in Fig. 807(7), cut by the plane *MNX*.

8-47-4. Cone in Fig. 807(8), cut by a vertical plane not parallel to the base or containing the vertex.

8-47-5. Cone in Fig. 807(8), cut by a sloping plane showing as an edge in the front elevation and not containing the vertex.

8-47-6. Cone in Fig. 807(8), cut by the plane *MNX*.

Group 48. Development of a Cone of Revolution

In each problem draw a right cone of revolution with the axis vertical. Assume it to be cut by the specified plane. Show the cut in both the given views and develop the lower or base end of the cone.

8-48-1. Plane *ABC*, Fig. 804(1).
8-48-2. Plane *ABCD*, Fig. 804(2).
8-48-3. Plane *ABC*, Fig. 804(3).
8-48-4. Plane *ABC*, Fig. 804(4).
8-48-5. Plane *ABC*, Fig. 804(5).
8-48-6. Plane *ABC*, Fig. 804(6).

Group 49. Development of an Oblique Cone with the Vertex Available

8-49-1. Develop the entire cone of Fig. 807(9).
8-49-2. Develop the entire cone of Fig. 807(10).

8-49-3. Develop the lower portion of the cone of Fig. 807(9) after it is cut by a vertical plane not parallel to the base nor containing the vertex.

8-49-4. Develop the lower portion of the cone of Fig. 807(10) after it is cut by a vertical plane not containing the vertex.

8-49-5. Develop the upper portion of the cone of Fig. 807(9) after it is cut by a sloping plane showing as an edge in the front view and not containing the vertex.

8-49-6. Develop the upper portion of the cone in Fig. 807(10) after it is cut by a sloping plane showing as an edge in the front view and not containing the vertex.

8-49-7. Develop the lower portion of the cone of Fig. 807(9) after it is cut by the plane MNX.

8-49-8. Develop the upper portion of the cone of Fig. 807(10) after it is cut by the plane MNX.

Group 50. Development of an Oblique Cone with the Vertex Unavailable

8-50-1. Develop the partial cone of Fig. 807(11).

8-50-2. Develop the partial cone of Fig. 807(12).

Group 51. Cone and Tangent Plane

In each problem show in the two given views a plane which is tangent to the given cone and contains the point X. Solve using only the two given views and check by a view showing the tangent plane as an edge.

8-51-1. Fig. 807 (7).

8-51-2. Fig. 807 (8). Extra view allowed.

8-51-3. Fig. 807 (9).

8-51-4. Fig. 807(10).

Group 52. Convolute

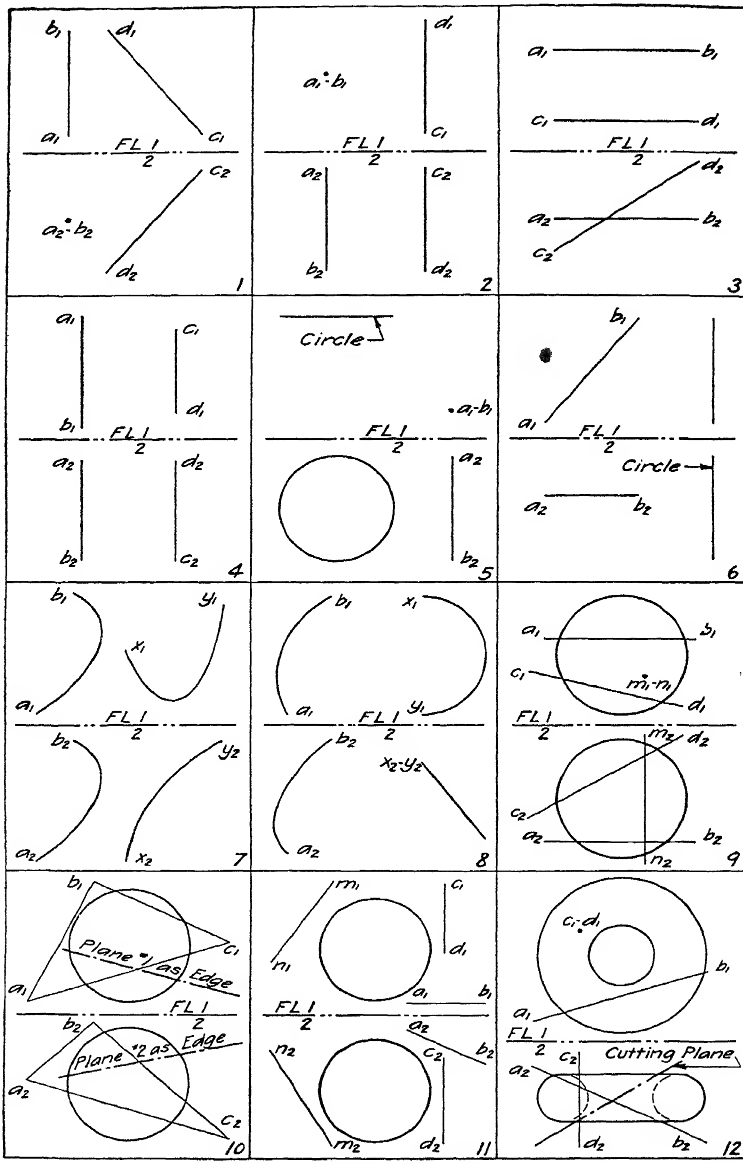
8-52-1. Draw the plan and front elevation of a convolute generated by a line remaining tangent to a left-hand cylindrical helix. The cylinder axis is vertical. Assume the lead equal to half the diameter. Show one complete sweep of the convolute extending to a level plane. Develop one complete turn. Calculate the number of turns that may be developed in one piece.

8-52-2. Draw the plan and front elevation of a convolute whose generatrix is tangent to a right-hand cylindrical helix. The axis of the cylinder is level. Assume the lead equal to the diameter. Show one complete turn extending to the plane of one of the bases. Develop one complete turn Calculate the number of turns that may be developed in one piece.

8-52-3. Use the same data as for problem 8-52-2. Let the convolute extend only to a larger cylinder twice as large in diameter as the first cylinder. Develop one complete turn of the portion of the convolute between the two cylinders.

Group 53. Helicoid

8-53-1. Draw two views of the helicoidal surface of a right-hand square thread. Assume the outside diameter about 2 in. and the lead about $\frac{3}{4}$ in. Show the thread for two complete turns.



FL 1/2
Circle
Circle
Plane 1 as Edge
Plane 2 as Edge
Cutting Plane
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 808.

8-53-2. Draw two views of the helicoidal surface of a triple right-hand square thread. Assume the outside diameter 2 in. and the lead $1\frac{1}{2}$ in. Show the triple threads for only one turn.

Group 54. Hyperbolic Paraboloid

In each problem draw three views of the hyperbolic paraboloid surface extending between the two given lines. The two lines may be extended at either end if it is necessary. The plane directrix will be specified in each case. Show about eight elements in each view.

8-54-1. Lines *AB* and *CD*, Fig. 808(1). Plane directrix is the front image plane.

8-54-2. Lines *AB* and *CD*, Fig. 808(2). Plane directrix is a level plane.

8-54-3. Lines *AB* and *CD*, Fig. 808(3). Plane directrix is a profile plane.

8-54-4. Lines *AB* and *CD*, Fig. 808(4). Plane directrix is a level plane.

8-54-5. Lines *AB* and *CD*, Fig. 804(8). Plane directrix is the front image plane.

8-54-6. Lines *AB* and *CD*, Fig. 804(10). Plane directrix is a level plane.

8-54-7. Lines *AB* and *CD*, Fig. 804(11). Plane directrix is a profile plane.

8-54-8. Lines *AB* and *CD*, Fig. 804(12). Plane directrix is a level plane.

Group 55. Conoid

8-55-1. Draw three views of a conoid surface connecting the circle and the line *AB* in Fig. 808(5). The plane directrix is level. Show eight elements in each view.

8-55-2. Draw three views of a conoid surface connecting the circle and the line *AB* in Fig. 808(6). The plane directrix is the front image plane. Show eight elements in each view.

Group 56. Cylindroid

8-56-1. Draw three views of a cylindroid surface connecting the two curved lines in Fig. 808(7). The plane directrix is level. Show eight elements in each view.

8-56-2. Draw three views of a cylindroid surface connecting the two curved lines in Fig. 808(8). The plane directrix is the front image plane. Show eight elements in each view.

Group 57. Hyperboloid of Revolution of One Sheet

In each problem draw the two simplest views of the hyperboloid of revolution of one sheet which would be generated by revolving the given line *CD* about the given line *AB* as an axis. Show twelve elements in each view.

8-57-1. Fig. 808(1).
8-57-2. Fig. 808(2).
8-57-3. Fig. 808(3).
8-57-4. Fig. 808(4).
8-57-5. Fig. 804(7).

Group 58. Sphere

In each problem find in both the given views the two points at which the specified line pierces the given sphere.

8-58-1. Line AB, Fig. 808(9).

8-58-2. Line CD, Fig. 808(9).

8-58-3. Line MN, Fig. 808(9).

In each problem show in both the given views the cut made by the specified plane on the spherical surface. Also show the true size of the cut.

8-58-4. Plane No. 1, Fig. 808(10).

8-58-5. Plane No. 2, Fig. 808(10).

8-58-6. Plane ABC, Fig. 808(10).

In each problem show in the two given views a plane that is tangent to the sphere and contains the specified line.

8-58-7. Line AB, Fig. 808(11).

8-58-8. Line CD, Fig. 808(11).

8-58-9. Line MN, Fig. 808(11).

Group 59. Torus

8-59-1. In Fig. 808(12) show in the two given views the points at which the line CD pierces the torus.

8-59-2. In Fig. 808(12) show in the two given views the points at which the line AB pierces the torus.

8-59-3. In Fig. 808(12) show in the plan view the sectional cut across the torus made by the given cutting plane.

Group 60. Double-curved Surfaces of Revolution

8-60-1. Draw three views of a paraboloid of revolution which is generated by revolving a parabola whose equation is $y^2 = 4x$. The axis of revolution is a symmetrical axis containing the focus. Calculate values of y for values of x from 1 to 6. Plot the curve to some scale. Assume the base to be a right section where x equals 6.

8-60-2. Draw three views of a paraboloid of revolution which is generated by revolving a parabola whose equation is $x^2 = 8y$. Take the axis to be a vertical line through the focus. Calculate values of x for values of y from 1 to 6. Plot the curve to some scale. Assume the base to be a right section where y equals 6.

8-60-3. Draw three views of a hyperboloid of revolution of two sheets which is generated by revolving a hyperbola whose equation is $(x^2/9) - (y^2/16) = 1$. Use a level axis through the foci.

Calculate values of y for values of x from 3 to 8. Plot the curve to some scale and make the bases right sections where x equals $+8$ or -8.

8-60-4. Same statement as for problem 8-60-3, but let the equation for the hyperbola be $(y^2/9) - (x^2/16) = 1$. Take the axis vertical.

8-60-5. Draw the three views of an ellipsoid of revolution generated by revolving an ellipse about its major axis. The equation of the ellipse is $(x^2/16) + (y^2/9) = 1$. Calculate the lengths of the major and minor axes and draw the ellipse by the trammel method. Take both axes to be level.

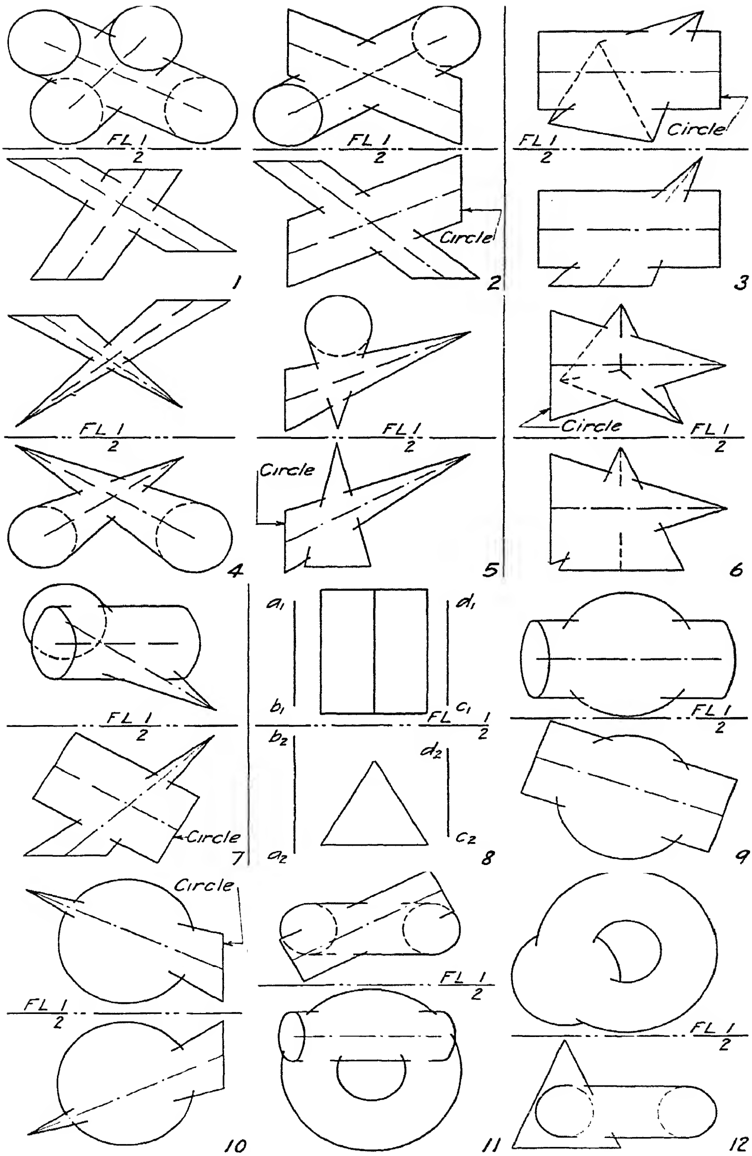


FIG. 809.

8-60-6. Same statement as for problem 8-60-5, but let the ellipse revolve about its minor axis.

8-60-7. An ellipsoid of revolution is generated by an ellipse having a major axis 3 units long and a minor axis 2 units long. The major axis is a vertical line. Draw three views of the surface.

Group 61. Intersection of Surfaces

In each problem find the line of intersection between the two given surfaces located as shown in the drawing. Show the hidden as well as the visible part of the curve and locate all special points of tangency. Show each extreme element with a solid line as far as it is visible.

8-61-1. Fig. 809(1). Two elliptical cylinders with axes non-intersecting. Also develop one of the cylinders and the line of intersection.

8-61-2. Fig. 809(2). Two elliptical cylinders. Also develop one of the cylinders and the line of intersection.

8-61-3. Fig. 809(3). Cylinder of revolution and oblique pyramid. Also develop both surfaces and show the line of intersection on each one.

8-61-4. Fig. 809(4). Two oblique cones. Show two possible methods for obtaining points on the curve of intersection. Also develop one of the cones and the line of intersection.

8-61-5. Fig. 809(5). Two oblique cones. Also develop one of the cones and the line of intersection.

8-61-6. Fig. 809(6). Cone of revolution and triangular pyramid. Also develop both surfaces and show the line of intersection on each one.

8-61-7. Fig. 809(7). Cylinder of revolution and oblique cone. Also develop both surfaces and show the line of intersection on each one.

8-61-8. Fig. 809(8). Triangular prism and hyperbolic paraboloid which connects the two given lines *AB* and *CD*. Also develop the prism and the line of intersection.

8-61-9. Fig. 809(9). Sphere and cylinder of revolution, axes non-intersecting. Also develop the cylinder and the line of intersection.

8-61-10. Fig. 809(10). Sphere and oblique cone, axes nonintersecting. Also develop the cone and the line of intersection.

8-61-11. Fig. 809(11). Torus and cylinder of revolution. Also develop the cylinder and the line of intersection.

8-61-12. Fig. 809(12). Torus and cone of revolution. Also develop the cone and the line of intersection.

Group 62. Locus

In each problem show in the given views the locus of the points or lines in the positions specified.

8-62-1. Fig. 803(11). Locus of all points in space equidistant from *A* and *B*.

8-62-2. Fig. 804(1). Locus of all lines in space having a slope of 15 degrees and making 15 degrees with plane *ABC*.

CHAPTER IX

DRAFTING-ROOM PROBLEMS

The problems in this chapter are all given with definite working data including the proper scale to use for each one. Practically all of them have engineering settings and a large number of them have been suggested by engineers who have had to solve similar problems in their own work. Every problem should be solved accurately for a result which should be scaled for a numerical value wherever possible. Required angles should be measured with a protractor to the nearest half of a degree, or the tangent of the angle should be measured. In all cases the results should be plainly marked.

The following fourfold experience will be gained by a careful solution of these problems.

1. Experience in laying out problems from data which closely resemble the data in an engineering office.
2. Experience in becoming familiar with engineering terms, in analyzing engineering situations, and in selecting the proper principles to be used in arriving at a correct solution.
3. Experience in establishing the habit of checking every problem by an independent method.
4. Experience in learning to work accurately in order to obtain proper results, and in learning to work clearly so that the drawing may easily be understood by others.

The problems are all laid out so that they can be solved on one of two standard-size sheets, using the scales that are given. The small sheet trims to a size 8½ by 11 in., and the large sheet trims to a size 11 by 16¼ in. There is a ¾-in. border on the left side of the large sheet and on the upper side of the small sheet, and a ¼-in. border on the other three sides of each sheet. The right edge of the large sheet will fold over to the left border line, and the sheet will file in a standard notebook with the small sheet. Both sheets are to be placed with the long edge parallel with the T square unless the problem number is followed by a *T*, which means to turn the sheet so the short edge is parallel with the T square. All problems are to be solved on small sheets unless the problem number is followed by an *L*, which means to use a large sheet. North is always at the top of the sheet unless it is otherwise stated.

Group 1. True Length and True Slope of a Line

9-1-1. Portion of concrete wing wall. Scale: ¼ in. = 1 ft. 0 in.

I. Find the true length of the corner *AB* in an inclined view.

II. Find the true slope of the corner *AB*.

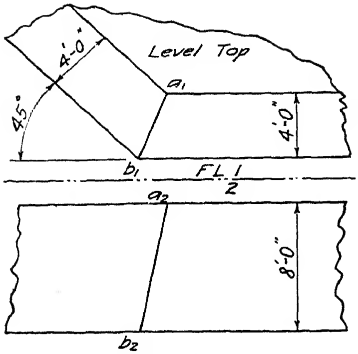


FIG. 9-1-1.

9-1-2. Portion of concrete pier. Scale: ¼ in. = 1 ft. 0 in.

I. Find the true length of the corner *AB* in an inclined view.

II. Find the true length of the corner *CD* in an elevation view.

III. Find the true slope of the corner *AB*.

IV. Find the true slope of the corner *CD*.

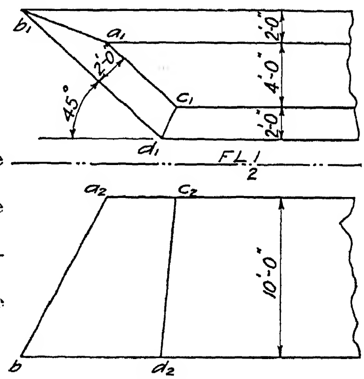


FIG. 9-1-2.

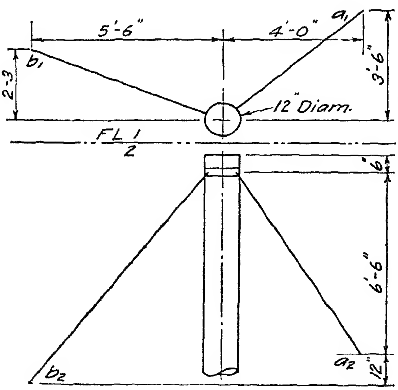


FIG. 9-1-3.

9-1-3. Two guy wires for a stack.

Scale: ½ in. = 1 ft. 0 in.

I. Find the true length of the wire to *A*.

II. Find the true length of the wire to *B*.

III. Find the true slope of the wire to *A*.

IV. Find the true slope of the wire to *B*.

V. Check both true lengths by new views.

9-1-4. Scale: 1 in. = 50 ft.

The points C and A are the portals of two tunnels. C is 200 ft. east and 50 ft. south of A and 50 ft. lower in elevation than A.

From A one tunnel bears N 60° E and falls 45 degrees (true slope).

From C the other tunnel bears N 45° W until it meets the tunnel from A.

Find:

I. The true slope of the tunnel from C.

II. The true length of the tunnel from C.

III. The true length of the tunnel from A.

9-1-5. Scale: 1 in. = 100 ft.

A tunnel 300 ft. long is driven from the point A bearing N 60° E and on a falling grade of 100 per cent. At what point on the map would you start a second tunnel bearing N 45° W and on a falling grade of 57.7 per cent so that it would meet the low end of the first tunnel? Assume that the two tunnels start at the same level.

Find:

I. The true length of the second tunnel.

II. The starting point of the second tunnel on the map with reference to A. Give coordinates.

9-1-6. Scale: 1 in. = 50 ft.

Two tunnels start from a common point A in a vertical shaft. Tunnel AB bears N 45° W, falls 15 per cent and is 150 ft. long. Tunnel AC bears S 75° W, falls 15 degrees and is 178 ft. long. The ends of the two tunnels are to be connected by a ventilating tunnel.

Find the following information regarding the ventilating tunnel.

I. The bearing.

II. The true length.

III. The per cent grade.

IV. The true slope in degrees.

9-1-7. A vertical mast, DE, and three guy wires to anchors A, B, and C.

Scale: ⅛ in. = 1 ft. 0 in.

Anchor A is 14 ft. W and 4 ft. N of DE and at an elevation of 140 ft.

Anchor B is 12 ft. E and 7 ft. N of DE and at an elevation of 145 ft.

Anchor C is 4 ft. E and 10 ft. S of DE and at an elevation of 138 ft.

Elevation of D = 158 ft. and of E = 143 ft. All elevations are referred to sea level. The boom is pivoted at E and is to swing clear around the mast at any angle of elevation.

Find:

I. The true length of each guy wire.

II. The length of the longest boom it is possible to use and have it clear all the guy wires.

III. The length of the longest horizontal boom that can be used.

IV. Keeping the anchor C on the same level, how far would it have to be moved due south so that a boom 9 ft. long could be used in any position?

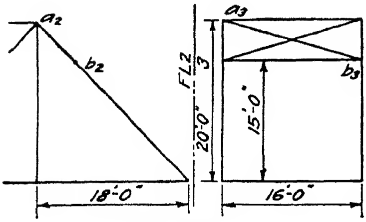


FIG. 9-1-8.

9-1-8. End panel of a bridge.
Scale: ⅛ in. = 1 ft. 0 in.
Two elevation views are given.
Find:
I. The true length of the diagonal member AB.
II. The true slope of the diagonal member AB.

Group 2. Edge View and True Size of a Plane

9-2-1. Plane ABC. Scale: ¼ in. = 1 ft. 0 in.
B is 4 ft. east and 7 ft. north of A and 5 ft. below A.
C is 10 ft. east and 4 ft. north of A and 3 ft. above A.
Find:
I. Edge view of the plane in an elevation view.
II. Edge view of the plane in an inclined view.
III. True slope of the plane.
IV. True size of the plane off each edge view.
V. Show in all views a point X on this plane which is 9 ft. away from C and at an elevation of 5 ft. below C.
VI. Show a level line on the plane through the point X in each true size view.

9-2-2. Scale: 1 in. = 40 ft.
AB and BC are the centerlines of two pipelines intersecting at B.
A is 60 ft. north and 60 ft. west of B and 60 ft. above B.
C is 30 ft. north and 120 ft. west of B and 20 ft. below B.
X is a point on the plane of ABC 40 ft. due west of A.
Find:
I. True size of plane angle ABC.
II. Where to connect into BC with a 45-degree connection for a straight pipe from X. Give the distance from B.
III. True length of the pipe from X to BC.
IV. True slope of the pipe from X to BC.
V. Show the pipe from X to BC in all views.

9-2-3. Skew Bridge. Scale: 3/32 in. = 1 ft. 0 in.
Only the end panel, $AEFC$, of this bridge is shown in Fig. 9-2-3.
Draw a view of this entire panel in which it will appear in its true size and shape. The members AC and EF are parallel.

9-2-4. Scale: ¼ in. = 1 ft. 0 in.
It is desired to make the shortest possible connection from a point C to a pipe AB which bears due north from A. The point B is 10 ft. due north of A and 8 ft. higher than A. The point C is 6 ft. due east of A and on the same level as B. Show the shortest connecting pipe in all views.
Find:
I. True length of this shortest pipe.
II. True slope of this shortest pipe.

III. True distance from A to the point of connection.
IV. Check by an independent method.

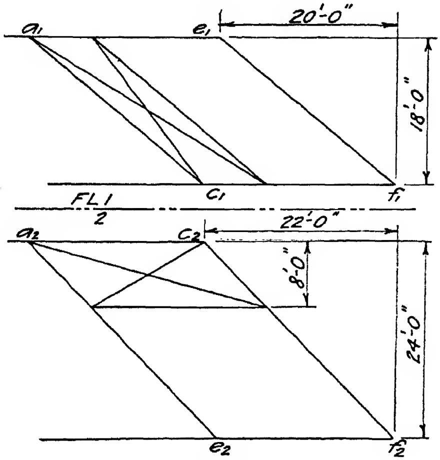


FIG. 9-2-3.

9-2-5. Scale: ¼ in. = 1 ft. 0 in.

A metal tank to be placed below deck on a ship has a level top, four vertical sides and a sloping bottom $ABCD$. AB and CD are not parallel but the bottom is a plane surface and C is on this plane. This fact may be used to locate C in the front elevation.

I. Draw an auxiliary elevation of the entire tank showing the bottom of the tank as an edge.

II. Draw an inclined view of the bottom of the tank only which will show it in its true size and shape.

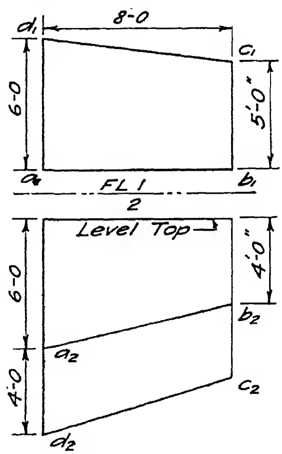


FIG. 9-2-5.

9-2-6. Plane surface ABC. Scale: ¼ in. = 1 ft. 0 in.

C is 9 ft. south and 4 ft. east of A and 8 ft. below A.

B is 6 ft. south and 10 ft. east of A and 2 ft. below A.

Show in the plan and front elevation the largest circle which may be laid out in the plane ABC and within the given limits of the plane.

Obtain the axes for each view direct from an edge view.

In a view showing the true size of the plane ABC show all four axes used for both views and label each one.

9-2-7.[L] Point of a concrete pier. Scale: 1/4 in. = 1 ft. 0 in.

The top and the bottom of the pier are level planes. Faces *A* and *B* have a true slope of 60 degrees. Halfway up the face *A*, on the centerline shown, is the center of a recess 6 ft. square and 18 in. deep. Two sides of the square opening are level. Halfway up the face *B*, on the centerline shown, is the center of a recess 6 ft. in diameter and 18 in. deep. Both recesses are set in at right angles to the sloping faces.

Show both the recesses in both the given views.

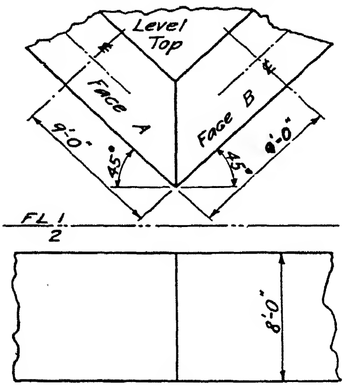


Fig. 9-2-7.

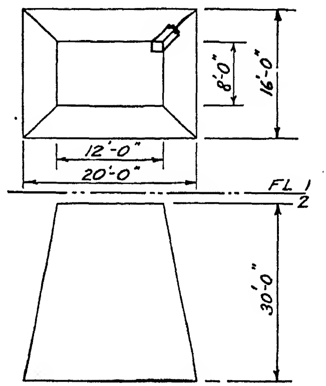


Fig. 9-2-8.

9-2-8. Timber tower supporting a tank. Scale for tower. 1/8 in. 1 ft. 0 in. Scale for angles. 1 1/2 in. = 1 ft. 0 in.

The four corner posts are 8 by 8 timber columns placed as shown in the sketch.

Find the angles of cut across the sides of the timber at the base which will make the end cut a level plane when the timber is in position.

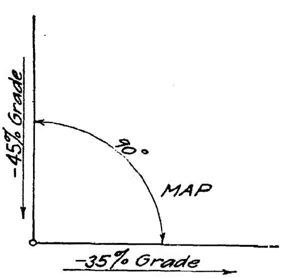


Fig. 9-2-9.

9-2-9. A water main having negative grades is to turn the corner at a 90-degree street intersection as shown on the map.

Find the angle required for the special elbow at this corner.

9-2-10. Steel penstock. Scale: 1 in. = 20 ft.

A, *B*, and *C* are three given points on the centerline of a 24-in. penstock through which water flows down a mountainside to the power house. *B* is 70 ft. west and 25 ft. north of *A*. *C* is 30 ft. west and 50 ft. north of *A* Elevations: *A* = 2,675 ft., *B* = 2,655 ft., *C* = 2,630 ft.

The pipe design calls for a long-radius elbow to connect the straight portions of the centerline in order to avoid the sharp turn at *B*. The specifications call for the radius of this elbow to be eight times the diameter of the pipe.

Find:

I. True size of the sweep angle of the elbow.
II. True length of straight pipe from A to the elbow.
III. True slope of straight pipe from A to the elbow.
IV. True length of straight pipe from the elbow to C.
V. True slope of straight pipe from the elbow to C.
VI. Show the centerline of the elbow in the plan and front elevation.

9-2-11. Scale: 1 in. = 40 ft.

A, B, and C are three points on the centerline of a steel penstock delivering water down a mountainside.

B is 130 ft. east and 20 ft. south of A and 20 ft. above A.

C is 50 ft. east and 80 ft. north of A and 70 ft. above A.

In order to avoid the sharp turn at A, a special elbow is to be installed there. This elbow is to come tangent to AB and AC, where it connects with them. The radius of the elbow is 60 ft. Find the bearing from A to the center of curvature of the elbow. Show the elbow in all views.

9-2-12. Scale: 1 in. = 60 ft.

The sketch shows some parallel straight-line contour lines on the ground, and two definitely located points, A and B.

I. Lay out the centerline of a road from A to B so that it will have a constant grade of 10 per cent for the entire distance. Use no more switch-backs than are necessary and neglect the curves at the turns.

II. What would be the maximum grade if a road of constant bearing were used?

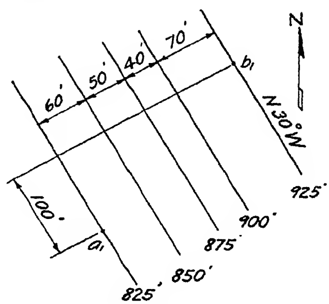


Fig. 9-2-12.

Group 3. Shortest Distance between Two Lines

9-3-1. Scale: 1 in. = 100 ft.

From A a new mining shaft bears N 30° E and falls 30 degrees.

From C an old mining shaft bears S 75° W and falls 15 degrees.

C is 140 ft. east and 210 ft. north of A, and at the same level as A.

It is desired to connect these two shafts with the shortest possible shaft. Use the line method.

Find:

I. True length of the connecting shaft.
II. True bearing of the connecting shaft.
III. True slope of the connecting shaft.

9-3-2. Using the same data and scale as for problem 9-3-1, make the solution by the plane method.

9-3-3. Scale: 1 in. = 30 ft. Two pipe lines, AB and CD.

B is 10 ft. north and 60 ft. west of A and 45 ft. above A.

C is 22 ft. due south of A and 75 ft. above A.

D is 50 ft. north and 35 ft. west of A and 35 ft. above A.

Show where to connect these two pipes with a third pipe, using only 90-degree tees. Find the true length and slope of the third pipe. Use the plane method.

9-3-4. Using the same data and scale as for problem 9-3-3, make the solution by the line method.

9-3-5. Scale: 1 in. = 200 ft. Two mining tunnels, *AB* and *CD*

B is 370 ft. south and 220 ft. east of *A* and 120 ft. above *A*.

C is 400 ft. south and 200 ft. west of *A* and 490 ft. above *A*.

D is 150 ft. north and 120 ft. east of *A* and 210 ft. above *A*.

It is desired to connect these two tunnels with the shortest possible shaft. Show this new shaft in all views.

Find:

I. True length of the new shaft.

II. True slope of the new shaft.

III. True bearing of the new shaft.

9-3-6. Scale: 1 in. = 40 ft.

Two mining tunnels are driven from the points *A* and *C*. The point *C* is 50 ft. east and 37 ft. south of *A* and 60 ft. lower than *A*. The tunnel from *A* bears due east and the one from *C* bears due north. Each tunnel is 100 ft. long and has a −63 per cent grade.

Using the line method, find the true length, slope and bearing of the shortest possible shaft to connect the two given tunnels.

9-3-7. Scale: 1 in. = 400 ft. Two mining shafts, *AB* and *CD*.

B is 800 ft. east and 600 ft. south of *A* and 800 ft. below *A*.

C is 100 ft. east and 900 ft. south of *A* and 600 ft. below *A*.

D is 700 ft. east and 100 ft. south of *A* and 100 ft. below *A*.

Connect these two shafts with the shortest possible level tunnel and show it in the plan and the front elevation.

Find:

I. True length of the level tunnel.

II. Bearing of the level tunnel.

9-3-8. Using the same scale and layout data as for problem 9-3-1, draw the two mining shafts and connect them with the shortest possible level tunnel.

Find:

I. True length of the level tunnel.

II. Bearing of the level tunnel.

9-3-9. Using the same scale and layout data as for problem 9-3-3, draw the two pipes and show how to connect them with the shortest possible level pipe.

Find:

I. True length of the level pipe.

II. Bearing of the level pipe.

9-3-10. Using the same scale and layout data as for problem 9-3-7, draw the two mining shafts and locate the shortest possible connecting shaft.

Find:

I. True length of the new shaft.

II. True slope of the new shaft.

III. Bearing of the new shaft.

9-3-11. Use the same layout data and the same scale as for problem 9-3-3. Find the location and the true length of the shortest connecting pipe to bear N 30° E and to have a rising 30 per cent grade.

Group 4. A Line Piercing a Plane

9-4-1. Scale: 1 in. = 40 ft.

A, *B*, and *C* are three points of outcrop on a vein of ore, and *XZ* is a tunnel.

B is 50 ft. east and 100 ft. north of *A*, and 70 ft. below *A*.

C is 130 ft. east and 50 ft. north of *A*, and 40 ft. above *A*.

X is 80 ft. due north of *A*, and 20 ft. above *A*.

Z is 60 ft. east and 10 ft. north of *A*, and 40 ft. below *A*.

Find where the tunnel pierces the vein, by three independent methods:

I. By using two views only and a vertical projecting plane.

II. By using two views only and a projecting plane showing as an edge in the front view.

III. By using the edge-view method.

9-4-2.[T] Scale ¼ in. = 1 ft. 0 in.

A rectangular-shaped bin with a sloping bottom, *ABCD*, which is pierced by a pipe.

Find where the centerline of the pipe pierces the bottom by three independent methods.

I. By using two views only and a vertical projecting plane.

II. By using two views only and a projecting plane showing as an edge in the front view.

III. By using the edge-view method.

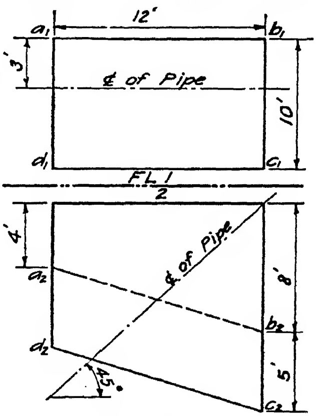


Fig. 9-4-2.

9-4-3. Scale: 1 in. = 20 ft.

A is a point of outcrop on a vein of ore which dips down 45 degrees toward the northeast. The upper and lower surfaces of the vein are parallel planes and the point *A* is on the upper plane. The strike of the vein is known to be N 45° W. *B* is 40 ft. east and 50 ft. north of *A*, and 5 ft. below *A*.

From *B* a shaft is driven bearing due west, and on a falling grade of 26.8 per cent. Measured along the centerline of this shaft, the distance through the vein is 20 ft. Find the real thickness of the vein.

9-4-4.[L] Scale: 1 in. = 100 ft.

A piece of sloping ground is represented by seven parallel straight-line contour lines bearing S 15° E and spaced 50 ft. apart on the map. The farthest one to the west is the 1,825-ft. contour, the next one toward the east is the 1,850-ft. line, the next one is the 1,875-ft. line, etc., the ground being a uniformly sloping plane all the way.

A tunnel, 250 ft. long, is driven from the point *A*, on the 1,825-ft. contour, bearing N 60° E on a falling slope of 45 degrees. At what point, *C*, on the map would you start a second tunnel to bear N 45° W from *C*, to fall

60 degrees, and to meet the low end of the first tunnel? Locate C on the map by coordinates from A.

Find:

I. Number of feet C is south of A.

II. Number of feet C is east of A.

III. True length of the tunnel from C.

9-4-5.[L] Scale: 1 in. = 50 ft.

The sketch shows a few contour lines on the surface of the ground. They are shown as parallel straight lines, just to simplify the drawing layout. Assume the ground to be a plane between any two contour lines. From the point A, on the 1,900-ft. contour line, the centerline of a road bears S 30° W, and it is to have a constant falling grade of 20 per cent. It will cross the valley on a fill, and it is to tunnel through the hill.

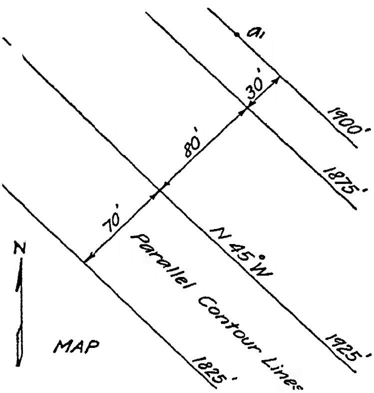


FIG. 9-4-5.

I. To the scale given, draw the plan view and an elevation view looking due north (front elevation). Place these views in about the center of the sheet.

II. Locate the point A near the upper border and show the centerline of the road in the same two views.

III. Using only the plan and front views, find the points where the centerline of the road would enter the hill and come out again. Locate these points first by using a vertical cutting plane, and then independently by using a cutting plane appearing as an edge in the front view. See that both methods check.

IV. Draw an edge view of the ground surface, locate the tunnel in this view, and check the piercing points again.

V. Scale the true length of the tunnel.

VI. Scale the distance from A to the tunnel.

VII. Scale the greatest depth of the fill over the 1,875-ft. contour line measured vertically.

Group 5. Intersection of Planes

9-5-1.[T] Scale: ¼ in. = 1 ft. 0 in. Two plane surfaces, ABC and XYZ both to be considered indefinite in extent.

B is 10 ft. west and 10 ft. north of A, and 8 ft. below A.

C is 15 ft. west and 3 ft. south of A, and 5 ft. above A.

X is 8 ft. west and 3 ft. north of A, and 5 ft. above A.

Y is 8 ft. east and 10 ft. north of A, and 3 ft. below A.

Z is 4 ft. east and 4 ft. south of A, and 8 ft. below A.

Using two views only, find four points on the line of intersection of the two planes. Find one point by each of the following methods.

I. By finding where the line *XZ* pierces the plane *ABC*.

II. By finding where the line *AC* pierces the plane *XYZ*.

III. By using any random vertical plane cutting both planes.

IV. By using any cutting plane appearing as an edge in the front view.

See that these four points check in a straight line in both views.

9-5-2. Scale 1/4 in. = 1 ft. 0 in. Two plane surfaces, *ABC* and *DEF*.
B is 5 ft. north and 6 ft. east of *A*, and 6 ft. below *A*.
C is 2 ft. south and 10 ft. east of *A*, and 3 ft. below *A*.
D is 2 ft. south and 1 ft. east of *A*, and 5 ft. below *A*.
E is 7 ft. north and 4 ft. east of *A*, and 1 ft. above *A*.
F is 4 ft. north and 12 ft. east of *A*, and 6 ft. below *A*.

Show the line of intersection of the two planes in both views by using the edge view of one of the planes.

9-5-3.[T] Scale: 3/8 in. = 1 ft. 0 in.

A, *B* and *C* are three located points on a sloping ground.
B is 13 ft. east and 2 ft. north of *A*, and 2 ft. above *A*.
C is 8 ft. east and 9 ft. south of *A*, and 4 ft. below *A*.

The vertical centerline, *XY*, of a pier is 7 ft. east and 2 ft. 6 in. south of *A*. The upper pier surface is level, and is 4 ft. above *A*. It is rectangular shaped and is 4 ft. wide from east to west and 3 ft. wide from north to south.

The sides of the pier have a batter of 1 to 4.

Using only the plan and the front elevation, show only that part of the pier that is above the ground. Solve each view independently and check by projection.

9-5-4. Scale: Actual size.

Draw the irregular-shaped pyramid shown in the sketch. Cut off the vertex of the pyramid by a plane in such a manner that the top of the remaining frustum will be a perfect parallelogram having two opposite sides each 1 in. long.

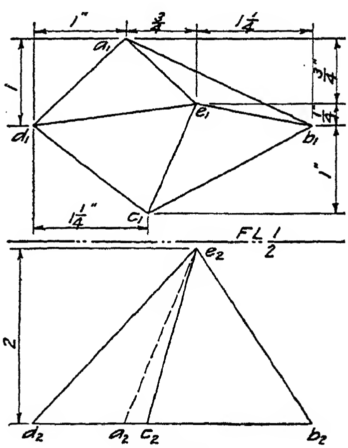


FIG. 9-5-4.

There are two possible solutions. (See Theorem 9, page 35.)

9-5-5. Assume two planes to be located about as shown in Fig. 9-5-5. Their slopes are fixed but their relative elevations are immaterial for this problem. Plane No. 1 appears as an edge in elevation view 2 and plane No. 2 appears as an edge in elevation view 3. Both planes may be considered to be indefinite in extent.

I. Show the line of intersection of the two planes in all views.

II. Draw a new view which will show both planes as edges.

9-5-6. Scale: Actual size.

Given two planes, *ABC* and *ADE*, located as shown in Fig. 9-5-6. Draw a new view in which both planes appear as edges. There are two distinctly

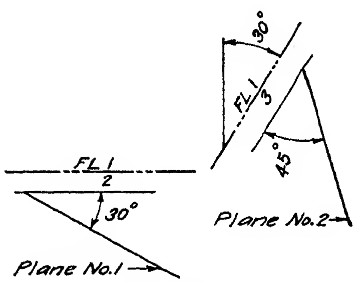


Fig. 9-5-5.

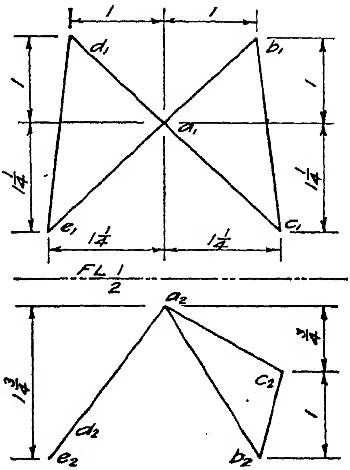


Fig. 9-5-6.

different methods. Solve by either method. Letter on the drawing a brief analysis of the other method.

Group 6. Line Perpendicular to a Plane

9-6-1.[L] Scale: 1 in. = 100 ft.

A, *B*, and *C* are points of outcrop on a vein of ore.

A is 450 ft. north and 280 ft. west of *B*, and its elevation is 2,490 ft.

C is 110 ft. north and 310 ft. east of *B*, and its elevation is 2,280 ft.

D is 420 ft. north and 150 ft. east of *B*, and its elevation is 2,560 ft.

The elevation of *B* is 2,680 ft.

From the point *D*, which is not on the vein, it is desired to drive the shortest possible tunnel to the vein.

I. Draw the plan and front elevation views of the vein (neglecting any thickness), as located by the three points of outcrop.

II. In these same two views show the required tunnel and find where it hits the vein, using no other views.

III. Also, using no other views, find the shortest possible level tunnel and find where it hits the vein.

IV. Check the piercing point of the shortest tunnel by locating it in an elevation view showing the vein as an edge.

V. Check the piercing point of the level tunnel by locating this tunnel in an inclined view showing the vein as an edge.

VI. Scale the true length of each tunnel from *D* to the vein and the grade of the shortest tunnel.

9-6-2. Scale: ¼ in. = 1 ft. 0 in. Plane surface *ABC*.

A is 11 ft. south and 9 ft. east of *B* and 5 ft. below *B*.

C is 8 ft. south and 7 ft. west of *B* and 10 ft. below *B*.

A horizontal line, not on the plane, starts at *C* and runs due east for 8 ft. Project this line on to the plane *ABC* and show its projection on this plane in the two given views. Use only two views.

9-6-3. Use the same scale and the same data for the plane as in problem 9-6-2. A level line bears due south from *B* for a distance of 8 ft. Project this line on to the plane *ABC* and show its projection on this plane in the two given views. Use only two views.

9-6-4.[T] Use the same bin as in problem 9-4-2 and draw it to the same scale. Assume the bin to be closed at the top by a level cover. Show the centerline of a pipe which extends only between the bottom and the cover of the bin. This pipe is perpendicular to the bottom plane of the bin at its central point. Show this centerline in both views, using no other views. Mark plainly the upper end of the pipe in both views.

9-6-5. Use the same data and the same scale as for problem 9-3-3. Show where to connect the two given pipes with a third pipe, using only 90-degree tees. The location of the third pipe must be made by using only the two given views. A new view may be drawn to find the true length and true slope of the third pipe.

9-6-6. Use the same data and the same scale as for problem 9-3-5. Show where to connect the two given tunnels with the shortest possible shaft, using only the two given views. After this shaft has been located, a new view may be drawn to find its true length and true slope.

9-6-7.[L] Scale: ¼ in. = 1 ft. 0 in. Plane surface, *ABC*.

B is 4 ft. south and 8 ft. east of *A* and 9 ft. above *A*.

C is 9 ft. south and 4 ft. west of *A* and 5 ft. above *A*.

I. At a point on the plane equidistant from *A*, *B*, and *C* erect a perpendicular above the plane and make it 6 ft. long. Show it in all views.

II. Locate this perpendicular by projection in an inclined view showing the plane as an edge. See that it checks at right angles to the plane.

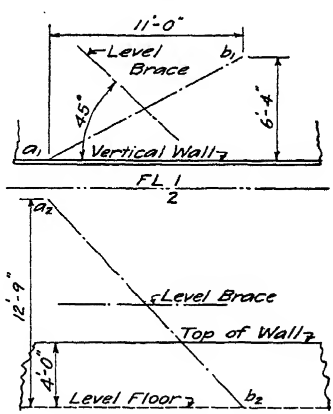


FIG. 9-6-8.

9-6-8. Scale: ¼ in. = 1 ft. 0 in.

The sketch shows a brace, *AB*, which is intersected at its center point by a level brace. From the point of intersection of these two braces it is desired to run a strut perpendicular to the plane of the two braces.

I. Using only the two given views, show the strut in these views.

II. Find how far below the top of the wall the strut would be fastened to the wall, using only two views.

III. Using a third view, find the true length of the strut.

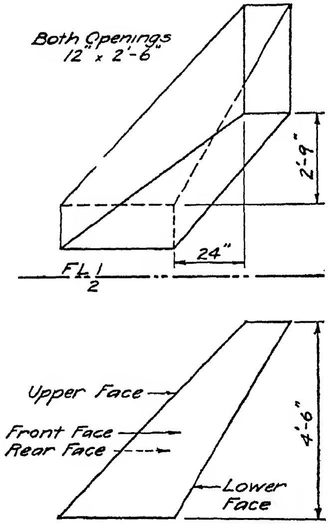


Fig. 9-7-4.

Group 7. Dihedral Angle

9-7-1. Use the same wingwall and the same scale as for problem 9-1-1.

Find the true size of the dihedral angle at the corner *AB*.

9-7-2. Use the same pier and the same scale as for problem 9-1-2.

Find the true size of the dihedral angle at the corner *AB*.

9-7-3. Use the same pier and the same scale as for problem 9-1-2.

Find the true size of the dihedral angle at the corner *CD*.

9-7-4. Scale: ½ in. = 1 ft. 0 in.

Offset piece made of steel plate.

Find the true size of the dihedral angle between the upper face and the front face.

9-7-5. Use the same offset and the same scale as for problem 9-7-4.

Find the true size of the dihedral angle between the rear face and the lower face.

9-7-6.[L] Hopper made of steel plate.

Layout scale: ¼ in. = 1 ft. 0 in. Detail scale: 3 in. = 1 ft. 0 in.

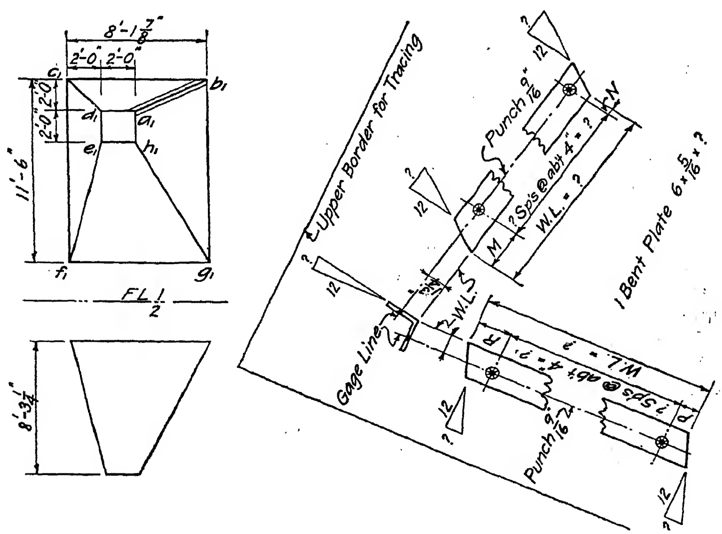


Fig. 9-7-6

The dimensions shown in the drawing are inside dimensions.

Adjacent side plates of the hopper are to be riveted to a special corner angle which is a bent plate and which is placed inside the hopper.

The problem is to detail the corner angle for the corner *AB*. Use a 5/16- by 6-in. plate of length to suit, and bend it lengthwise in the middle to fit flush inside the hopper. The top and bottom ends are to be flush with the edges of the hopper. Use ½-in. rivets on about 4-in. centers on a 1¾-in. gage, and countersunk on the inside. Make a completely dimensioned steel-shop drawing of this angle ready for use, *i.e.*, bent, punched, and trimmed.

Method of procedure:

I. Layout work. On the left side of the sheet show the plan and the front elevation of the hopper, and solve for the dihedral angle at *AB* and the end cuts. Do this in such a manner that these angles may be easily transferred to the shop drawing.

II. Mathematical work. Calculate the true length of the work line to the nearest 1/16 in. and express the bevel angles by their tangent with 12 as one leg (see Fig. 207).

III. Detailing work. On the right side of the sheet start the detail drawing, keeping it in the same position on the paper as the layout solution occupies, as in Fig. 9-7-6. Complete the detail drawing and show all necessary dimensions. Figure 9-7-6 shows a method of dimensioning which would be satisfactory to a steel-fabricating shop. The dimensions marked with a question mark must be determined. The dimensions marked *M*, *N*, *P*, and *R* must be such that the center of the end rivet is 1½ in. from the nearest sheared edge of the plate.

IV. Tracing work. Trace the detail drawing only, squaring it up with the border as shown in the figure.

9-7-7.[L] Use the same hopper, the same scales, and the same specifications as for problem 9-7-6. Make the solution for the corner *CD*.

9-7-8.[L] Use the same hopper, the same scales, and the same specifications as for problem 9-7-6. Make the solution for the corner *EF*.

9-7-9.[L] Use the same hopper, the same scales, and the same specifications as for problem 9-7-6. Make the solution for the corner *GH*.

9-7-10.[L] Steel-plate hopper. The dimensions shown are outside dimensions.

Layout scale: ½ in. = 1 ft. 0 in. Detail scale: 3 in. = 1 ft. 0 in.

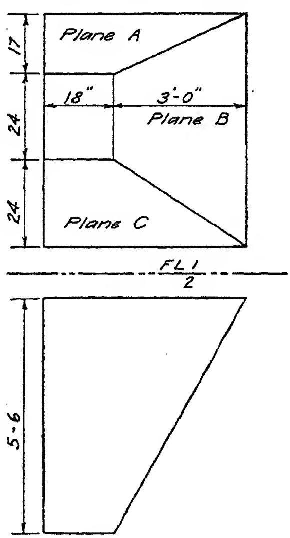


Fig. 9-7-10.

Make a completely dimensioned detail shop drawing of the riveted bent-plate connection between the plates for plane *A* and plane *B*.

The bent plate is to go on the outside of the hopper and is to fit flush at the top and bottom edges of the hopper.

Read problem 9-7-6 and follow the method of procedure suggested there.

9-7-11.[L] Use the same data and the same scales as for problem 9-7-10. Make the same solution for the plates between plane *B* and plane *C*.

9-7-12.[L] Use the same data and the same scales as for problem 9-7-10. Make the same solution for the plates between plane *C* and the vertical side.

9-7-13.[L] Use the same data and the same scales as for problem 9-7-10. Make the same solution for the plates between plane *A* and the vertical side.

Group 8. Angle between a Line and a Plane

9-8-1.[L] Solution scale: ½ in. = 1 ft. 0 in. Detail scale: 3 in. = 1 ft. 0 in. The figure shows two views of a plane to which a casting must be fastened for anchoring a strut from the point *X*. The centerline of the strut is as shown and it slopes down 40 degrees as it leaves *X*. It is not necessary to draw the front elevation.

I. Find the centerline length of the strut from *X* to where it meets the plane.

II Find the angle *N*, in degrees, which the strut makes with the plane.

III. Find the angle *M*, in degrees, which the centerline appears to be away from the 6-in. side of the casting base, when looking perpendicular to the plane. The 6-in. side is to be parallel to *AB* on the plane.

IV. Make a detailed shop drawing of the casting similar to the one shown, for receiving this strut. The base of the casting is 4 by 6 in., and the metal is ¼ in. thick all over. The receiving socket should have a hole ½ in. in diameter with a clear depth of 3 in.

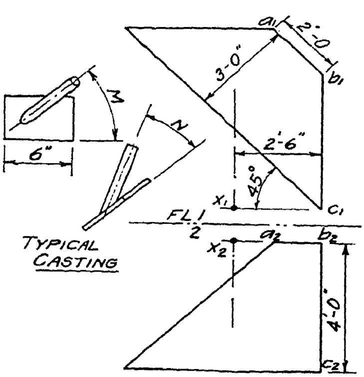


FIG. 9-8-1.

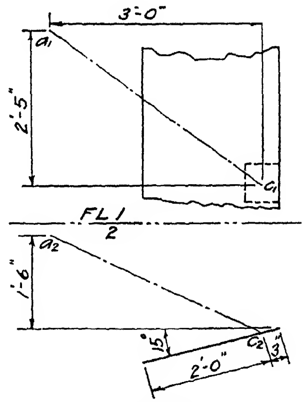


FIG. 9-8-2.

9-8-2.[T] Scale: 1 in. = 1 ft. 0 in.

The figure shows an approximation of a portion of a wing on an airplane to which the strut *AC* is to be fastened at *C*. Design the necessary fitting

for receiving the strut similar to the one shown in Fig. 9-8-1. The dashed lines in the plan indicate how the base of the casting is to be placed.

Make the solution for the angles M and N.

9-8-3.[L] Scale: $\frac{1}{8}$ in. = 1 ft. 0 in. Steel stack running up the outside of a building and braced to a parapet wall. Figure 9-8-3 shows the plan view of an actual installation which was recently made at a Seattle plant.

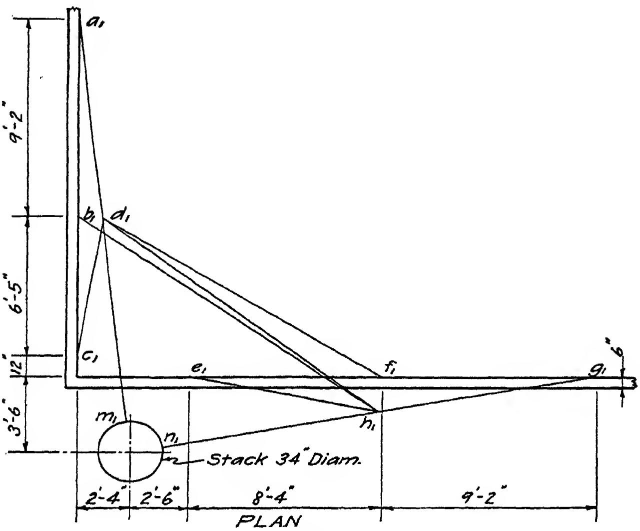


Fig. 9-8-3.

The points A, B, C, E, F, and G are on the inside top edge of the parapet wall. The points M and N are on a steel ring 2 ft. below the top of the stack. The top of the stack is 30 ft. 2 in. above the top of the wall. The points D and H are the center points of the long braces AM and GN, respectively, whose centerlines intersect the center of the stack. The brace DH is level. From D one brace extends to each wall at C and F. From H one brace extends to each wall at B and E. These braces were all starred angle sections with bent-plate connections fastened to the wall on the inside. Only the centerlines or worklines need to be dealt with here and it is not necessary to draw any front elevation.

Find:

I. The angle for the bent-plate connection at A.

II. The angle for the bent-plate connection at B.

III. The angle for the bent-plate connection at C.

9-8-4.[L] Use the same data and scale as for problem 9-8-3.

Find:

I. The angle for the bent-plate connection at E.

II. The angle for the bent-plate connection at F.

III. The angle for the bent-plate connection at G.

9-8-5. Scale: ¼ in. = 1 ft. 0 in.

An A frame, *ABC*, and two guy wires fastened to the top point *C*.

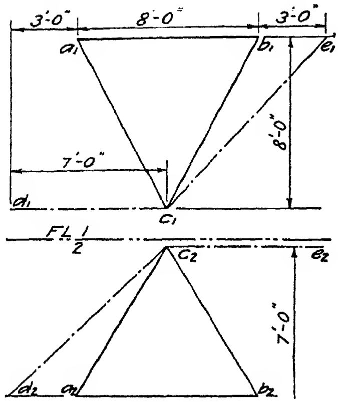


FIG. 9-8-5.

Find the true size of the angle between the guy *CE* and the plane of the frame.

9-8-6. Use the same data and scale as for problem 9-8-5.

Find the true size of the angle between the guy *CD* and the plane of the frame.

Group 9. Revolution

9-9-1. Scale: Actual size.

A level line *AB* is 3 in. long and bears S 45° E from *A*.

Point *X* is 2 in. due south of *A* and 1 in. higher than *A*.

Revolve the point *X* about the line *AB* as an axis until it lies at an elevation ½ in. lower than *AB*. Call the new position of the point *X′* and show it in the plan and the front elevation. There are two solutions.

9-9-2.[L] Scale: ½ in. = 1 ft. 0 in.

AB is the centerline of a shaft which makes an acute angle with the vertical side wall and the level base of a machine. A crank 27 in. long is at right angles to the shaft *AB* at the midpoint of the shaft.

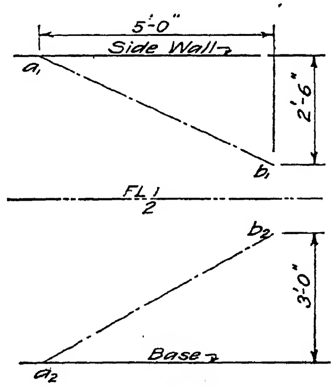


FIG. 9-9-2.

I. In the plan and the front elevation show the crank in the following positions with dash lines:

1. As it hits the base.
2. As it hits the side wall.
3. When it occupies its highest position.

II. What is the maximum number of degrees through which the crank can rotate?

III. What is the maximum length the crank could be and just clear the wall?

IV. What is the maximum length the crank could be and just clear the floor?

V. How far will the side wall have to be moved back to allow the 27-in. crank to rotate through exactly 180 degrees?

Tabulate the results for II, III, IV, and V and indicate clearly on the drawing where each result was measured.

9-9-3. Use the data and the scale for the shaft *AB* in problem 9-9-2.

Divide a small sheet into two parts by a line parallel to the short edge of the paper. Draw two views of the shaft only in each half of the sheet.

In one half of the sheet revolve the shaft AB until it shows in its true length in the plan.

In the other half of the sheet revolve the shaft AB until it shows in its true length in the front elevation.

Show and label the axis of revolution used in all four views.

See that the true lengths obtained by both methods check.

Mark the true slope of the shaft where it is seen.

9-9-4. A steel tank with a flanged elbow connection.

Flange B on the elbow is to have bolt holes drilled in it so that, when it is bolted to flange A on the tank, the straight pipe will have a 66.7 per cent downward slope to the right.

Flange A is 1 in. thick and 6 by 11 in. in diameter, and has eight ⅞-in. holes on a 9½-in. bolt circle.

It is standard practice for the bolt holes to straddle the centerline as shown on flange A. It will be necessary to locate on flanged B the centerline which the holes will straddle so that the given condition will be satisfied. The elbow is a standard 45-degree elbow with an undrilled flange.

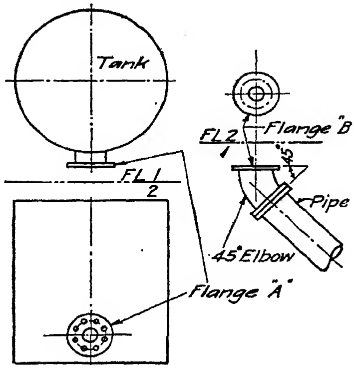


Fig. 9-9-4.

I. Find the angle between the elbow centerlines as shown and the centerlines to be used for drilling bolt holes.

II. Find the bearing of the pipe after it is installed, assuming that flange A faces south.

III. Complete the drawing of flange B, scale 1½ in. = 1 ft. 0 in., showing the holes properly located.

Note: It is not necessary to draw the tank. Use centerlines only.

9-9-5. Scale: ¼ in. = 1 ft. 0 in. Plane surface ABC.

B is 2 ft. east and 7 ft. north of A and 6 ft. below A.

C is 10 ft. east and 4 ft. north of A and 1 ft. above A.

I. Revolve the plane ABC until it shows in its true size in the plan.

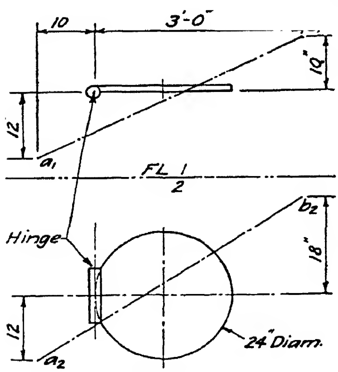


Fig. 9-9-6.

II. Revolve the plane ABC until it shows in its true size in the front elevation.

III. Check the true size by an inclined view.

9-9-6.[T] Scale: 1½ in. = 1 ft. 0 in.

AB is the centerline of a rod passing through the disk which is hinged on the left side as shown. The disk can move forward only through an angle of

120 degrees. Show the slot which must be cut in the disk to allow it to have this motion.

9-9-7. Scale: ¼ in. = 1 ft. 0 in.

A, *B*, and *C* are points of outcrop on a body of ore.

B is 10 ft. north and 5 ft. east of *A* and 7 ft. below *A*.

C is 5 ft. north and 13 ft. east of *A* and 4 ft. above *A*.

Find the true dip of the vein, using only the plan and front elevation views.

9-9-8. Scale: 1 in. = 40 ft.

A, *B*, and *C* are three located points on a plot of ground.

B is 100 ft. north and 50 ft. east of *A*, and 70 ft. below *A*.

C is 50 ft. north and 130 ft. east of *A*, and 40 ft. above *A*.

Find the centerline of a road from *A* having a 20 per cent rising grade. Show it in the plan and front elevation, using only these two views.

9-9-9. Scale: ¼ in. = 1 ft. 0 in.

A level pipe bearing due south projects one foot out from a vertical wall running east and west, and is 8 ft. above a level floor. A one-sixth bend (120-degree angle) is screwed on to the end of this pipe. A pipe screwed into this bend makes an angle of 40 degrees with the floor and just touches the floor. Using only the plan and the front elevation, show the pipe in these two views.

Find:

I. The true length of the pipe.

II. The distance the lower end of the pipe is out from the wall.

9-9-10. Use the same data and the same scale as for problem 9-8-5. By the method of revolution find the true size of the angle the guy *CE* makes with the plane of the frame.

9-9-11. Use the same data and the same scale as for problem 9-8-2. By the method of revolution find the true size of the angle between the strut *AC* and the wing.

9-9-12. Use the same data and the same scale as for problem 9-7-4. By the method of revolution find the true size of the dihedral angle between the upper face and the rear face.

9-9-13. Use the same data and the same scale as for problem 9-7-10. By the method of revolution find the true size of the dihedral angle between the two planes *A* and *B*.

9-9-14. Use the same data and scale as for problem 9-5-6. Revolve the plane *ADE* until the dihedral angle between the two planes at *A* is exactly 45 degrees. The line of intersection of the two planes must remain unchanged during the revolution.

Group 10. Noncoplanar Structures and Vectors

The problems in this group are all to be solved by the graphical method. Hard and well-sharpened pencils must be used in order to obtain accurate results.

9-10-1.[L] Space scale: ½ in. = 1 ft. 0 in.

Force scale: To be selected.

A hanging structural-steel frame of three members which support a given load. Find the load on each member. Use Bow's notation in the plan view.

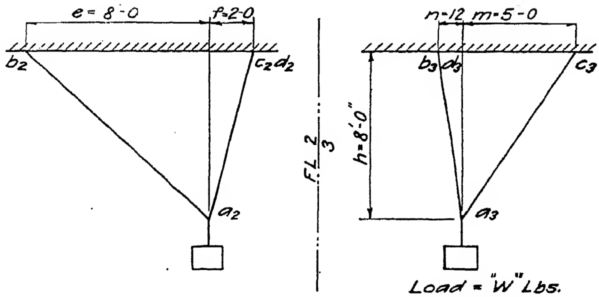


FIG. 9-10-1.

9-10-2.[L] Use the same scales and the same frame as for problem 9-10-1 but substitute the following values in the space drawing.

h = 10 ft. 0 in.
e = 6 ft. 0 in.
f = 2 ft. 0 in.
n = 2 ft. 0 in.
m = 5 ft. 0 in.

Solve for the load on each member.

9-10-3.[L] Use the same scales and the same frame as for problem 9-10-1 but substitute the following values in the space drawing.

h = 8 ft. 0 in.
e = 6 ft. 0 in.
f = 3 ft. 0 in.
n = 4 ft. 0 in.
m = 2 ft. 0 in.

Solve for the load on each member.

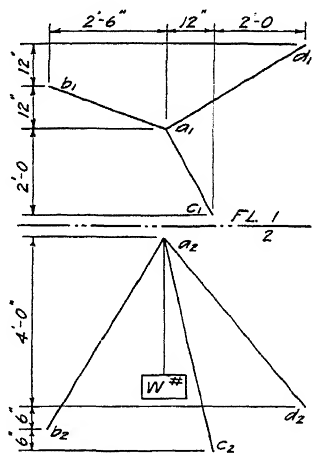


FIG. 9-10-4.

9-10-4.[L] Space scale: 1 in. = 1 ft. 0 in.
Force scale: To be selected.
A steel-frame tripod of three members and supporting a given load. Find the load on each member which is caused by the given load.

9-10-5.[L] Space scale: $\frac{3}{16}$ in. = 1 ft. 0 in.
Force scale: To be selected.
A derrick with a boom elevated at an angle of 30 degrees and supporting a given load at its end.
Find the load on
I. The boom AC.
II. The tie BC.

III. The mast AB.
IV. The two guys BD and BE.

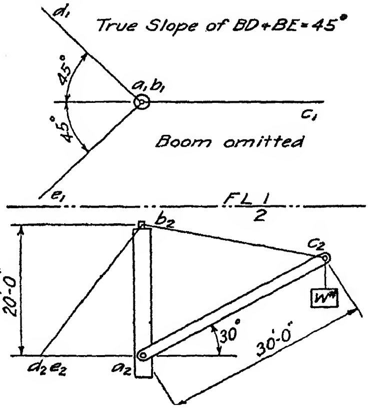


FIG. 9-10-5.

9-10-6.[L] Space scale: 1 in. = 10 in.
Force scale: 1 in. = 400 lb.

The cabane system of support between the wing and the fuselage of an airplane. A known horizontal load and a known vertical load are acting at A.

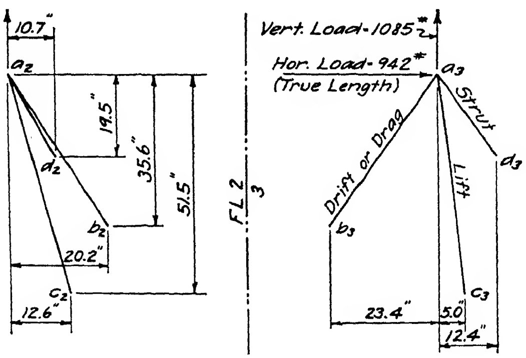


FIG. 9-10-6.

Draw the plan and front elevation of the three members and the loads. Draw the vector diagram for the five noncoplanar forces acting at A.

Find the value of the force acting in each of the three members AB, AC, and AD. Indicate whether each load is a tension or a compression load.

9-10-7. Vector scale: 1 in. = 10 ft. per second.

Three vectors representing velocities have the directions in space as shown in the sketch. The values of these velocities are as follows:

$DA = 24$ ft. per second.
$DB = 18$ ft. per second.
$DC = 12$ ft. per second.

Using only the two given views draw the vector diagram and find

I. The value of the resultant velocity.

II. The true slope of the resultant velocity.

III. The direction of the resultant velocity in the plan. Give the angle between its plan view and d_1a_1.

9-10-8. Vector scale: 1 in. = 30 ft. per minute.

Three velocity vectors are fixed in space as shown in the sketch. The values of the velocities are:

$AD = 60$ ft. per minute.
$AC = 113$ ft. per minute.
$AB = 134$ ft. per minute.

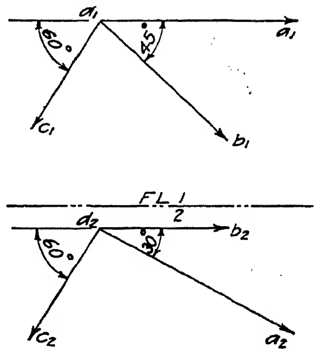


Fig. 9-10-7.

Using only the two given views draw the vector diagram and find:

I. The value of the resultant velocity.

II. The slope of the resultant velocity.

III. The direction of the resultant velocity in the plan. Give the angle from a_1b_1 or a_1c_1.

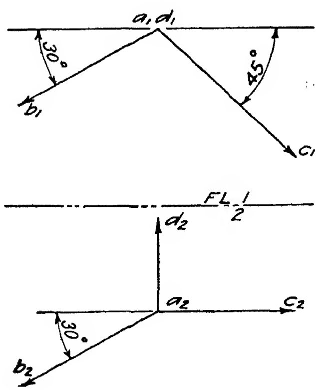


Fig. 9-10-8.

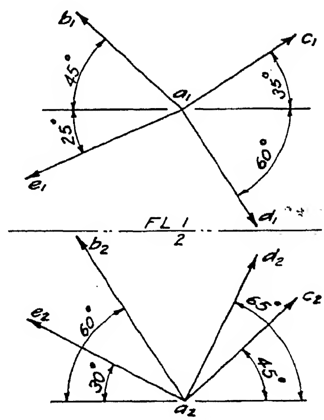


Fig. 9-10-9.

9-10-9. Vector scale: 1 in. = 20 ft. per second.

Four velocity vectors are fixed in space as shown in the sketch. The true values of the four velocities are:

AB	50	ft. per second.
AC	38.5	ft. per second.
AD	47	ft. per second.
AE	43	ft. per second.

Using only the given views draw the vector diagram and find:

I. The value of the resultant velocity.

II. The slope of the resultant velocity.

III. The direction of the resultant velocity in the plan referred to a_1c_1.

9-10-10. Use the same scale and the same data as for problem 9-10-4. On a large sheet make the solution by the method of Section 9.5.

9-10-11. Use the same scale and the same data as for problem 9-10-1. On a large sheet make the solution by the method of Section 9.5.

Group 11. Cylinders

9-11-1. Scale: Actual size. An oblique cylinder and a line AB.

Draw an oblique cylinder having an axis 3 in. long, bearing N 45° W and sloping up 45 degrees. Both upper and lower bases are level circles 1½ in. in diameter.

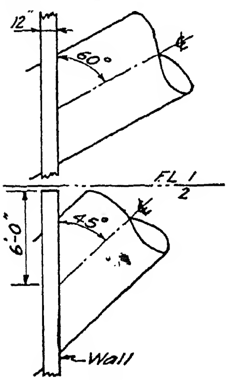


FIG. 9-11-2.

Line AB. A is 1½ in. due west and ¾ in. above the center of the lower base. B is 1¾ in. east and 1½ in. north of A, and 1⅜ in. above A.

Using two views only, find the two points where the line AB pierces the cylinder. Check these points by drawing a view showing the cylinder as an edge.

9-11-2. Scale: ¼ in. = 1 ft. 0 in.

A steel penstock passes through the wall of a powerhouse. The outside diameter of the penstock is 6 ft. Find the true size of the opening to allow in the wall. Show the opening on the near side of the wall only.

9-11-3. Scale: ¼ in. = 1 ft. 0 in.

In Fig. 9-11-2 suppose the hole in the wall is round and 6 ft. in diameter. Find the true shape the right section of a pipe would have to be to pass through the hole. Show a line on the developed pipe where a flange could be welded to the pipe so that the flange would bear against the wall all the way around.

9-11-4. Scale: 1½ in. = 1 ft. 0 in.

From a given point, A, the centerline of a flanged pipe bears N 45° E and rises with a true slope of 30 degrees. The pipe has an inside diameter of 12 in., an outside diameter of 13 in., and an overall length of 16 in. On the far end is a flange ¾ in. thick and 17 in. in diameter.

Show the pipe in the plan and front elevation views. Show only four 1 3/16-in. bolt holes through the flange and spaced 90 degrees apart around the flange on a bolt circle 15 in. in diameter.

9-11-5. Scale: 1 in. = 1 ft. 0 in.

A steel cylindrical shell has three cuts across its surface as shown in the sketch.

Make the complete development of the portion of the shell shown.

9-11-6. Scale: 1½ in. = 1 ft. 0 in.
A cylindrical pipe is shown having two angular cuts at the ends. Develop the portion of the pipe which is shown.

9-11-7.[L] Scale: 1 in. = 1 ft. 0 in.
The line AB is a portion of the centerline of a pipe 18 in. in diameter. The point A lies in the level floor and the point B lies in the vertical wall.

I. Show the plan and front elevation of the portion of the pipe which lies between the floor and the wall.

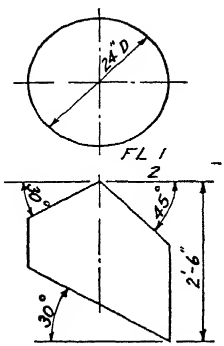


FIG. 9-11-5.

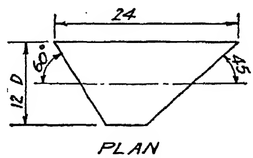


FIG. 9-11-6.

II. Assuming the pipe is to be cut from a stock length, find the angle of twist between the two end cuts.

III. Show the development of a piece of pipe-covering to cover the portion of the pipe lying between the floor and the wall.

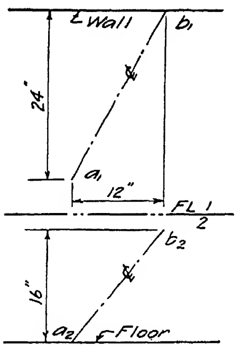


FIG. 9-11-7.

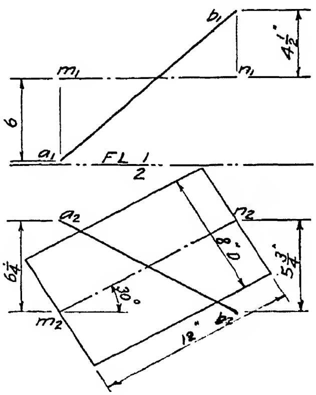


FIG. 9-11-9.

9-11-8.[L] Use the same scale and the same layout for the line AB as for problem 9-11-7. Let the pipe be only 15 in. in diameter and let the floor be inclined with a rising 20 per cent grade in going from A to the wall. Point A lies in this inclined floor and point B is still the same height above A.

Solve questions I, II, and III of problem 9-11-7 for this changed condition.

9-11-9.[T] Scale: 3 in. = 1 ft. 0 in.
The line MN is the axis of a cylinder which is shown in the front elevation only in the sketch.
Using only these two given views, find the two points where the line AB pierces the cylinder.

Check these two points by the edge-view method.

9-11-10. Scale: 3 in. = 1 ft. 0 in.

Draw the plan and the front elevation of the cylinder and the point A as they are located in Fig. 9-11-9.

I. In the two given views show two planes, both of which contain the point A and are tangent to the cylinder.

II. Find the true slope of both of these tangent planes.

Group 12. Cones

9-12-1. Scale: Actual size.

Draw a right cone of revolution with a base 3 in. in diameter and a 3½-in. altitude. The axis is vertical. The vertex of the cone is V.

Line AB. A is 2 in. due west of V and 2¼ in. below V. B is 1 in. east and ½ in. north of V and 1 in. below V.

Using only the plan and front elevation views, find the two points where the line AB pierces the cone and show them in both views.

9-12-2.[L] Scale: Actual size. Refer to Fig. 610.

Assume the cone in this figure to have a vertical axis 3½ in. long and a level base 3 in. in diameter. Cut this cone by the four planes B, C, D, and E as shown, assuming the plane B to be 1½ in. below the vertex of the cone.

Show each cut across the cone in the plan view and also in its true size.

9-12-3. Scale: 1 in. = 1 ft. 0 in.

The sketch shows a hopper made of steel plate. The axis is vertical. Complete the plan view and show the development of the plate.

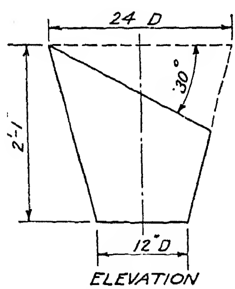


FIG. 9-12-3.

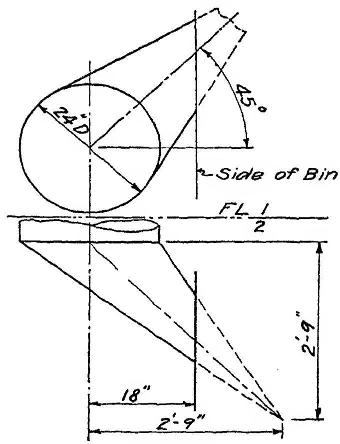


FIG. 9-12-4.

9-12-4. Scale: 1 in. = 1 ft. 0 in.

The sketch shows a hopper made of steel plate and opening into the vertical side wall of a bin. Develop the plate for the hopper. Show the true shape of the hole to be cut in the side wall of the bin.

9-12-5.[L] Scale: ½ in. = 1 ft. 0 in.

A conical-shaped hopper connecting two round pipes. Develop the steel plate for making this hopper.

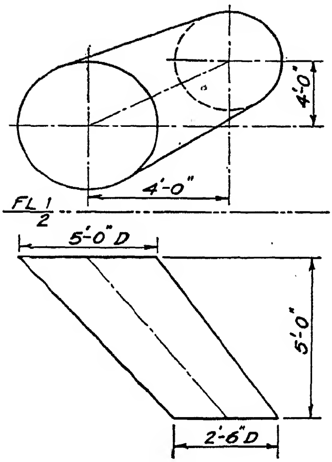


FIG. 9-12-5.

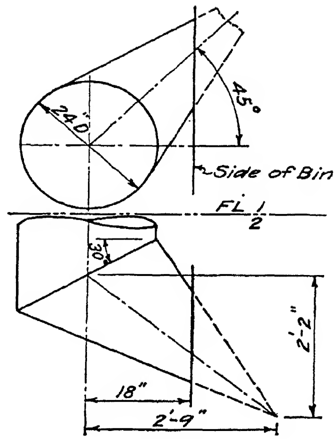


FIG. 9-12-6.

9-12-6. Scale: 1 in. = 1 ft. 0 in.

The sketch shows a hopper made of steel plate and opening into the vertical side wall of a bin. Develop the plate for the hopper. Show the true shape of the hole to be cut in the side wall of the bin.

9-12-7. Scale: ¾ in. = 1 ft. 0 in.

A given cone of revolution with a level axis and a point X not on the cone.

I. Show the two possible planes which may be drawn tangent to the cone and containing the point X.

II. Find the true slope of each of these tangent planes.

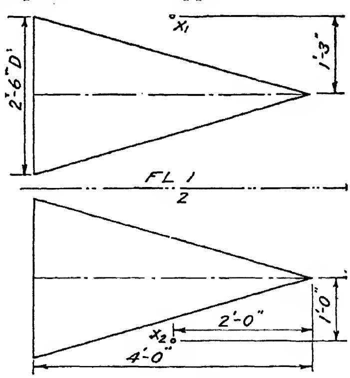


FIG. 9-12-7.

9-12-8. Scale: ¼ in. = 1 ft. 0 in.

Draw a right cone of revolution having a vertical axis 12 ft. high and a base 12 ft. in diameter.

Locate a point X so that it is 12 ft. east, 1 ft. south, and 6 ft. below the vertex of the cone.

Using only the plan and front elevation views, show a plane which is tangent to the cone and which contains the point X. There are two possible solutions.

9-12-9.[T] Scale: ⅛ in. = 1 ft. 0 in.

A line AB is the centerline of a pipe line. A point C is not on the line AB.

B is 18 ft. east and 10 ft. south of *A* and 19 ft. below *A*.

C is 6 ft. east and 19 ft. south of *A* and 19 ft. below *A*.

I. Show how to connect *C* with the pipe *AB*, using a pipe having the same grade as the pipe *AB*.

II. Find the true length of this connecting pipe.

Group 13. Spheres

9-13-1. Scale: Actual size.

Draw any three orthographic views of a sphere 3 in. in diameter. Point *C* is the center of the sphere and points *A* and *B* are two points on the surface of the sphere. In the plan view, *A* is $\frac{3}{4}$ in. to the left and $\frac{1}{2}$ in. in front of *C*. Also the point *B* is $1\frac{1}{4}$ in. in front and $\frac{1}{2}$ in. to the right of *C*.

Show both points *A* and *B* on the sphere in all three views.

Show the true length of the shortest distance from *A* to *B* measured on the surface of the sphere.

9-13-2. Scale: Actual size.

Draw the plan and front elevation views of a sphere 2 in. in diameter. The center of the sphere is *C*. Locate the line *AB*.

A is $1\frac{1}{4}$ in. south and $\frac{1}{4}$ in. west of *C*, and $1\frac{1}{2}$ in. above *C*.

B is 1 in. east and 1 in. north of *C*, and $\frac{3}{4}$ in. below *C*.

Find the two points where the line *AB* pierces the sphere.

I. By the great-circle method.

II. By the small-circle method.

9-13-3. Scale: Actual size.

Three spheres, whose diameters are $1\frac{1}{2}$ in., in., and 2 in., respectively, have their centers in the same level plane. They are also tangent to each other. A fourth sphere, whose diameter is in., is tangent to all three of the given spheres.

Show all four spheres in the plan and in a side elevation view.

Dash the invisible part of each sphere.

9-13-4. Scale: Actual size.

Three spheres, whose diameters are $1\frac{1}{2}$ in., $1\frac{3}{4}$ in., and 2 in., respectively, are resting on a level plane. A fourth sphere $2\frac{1}{2}$ in. in diameter rests on top of the other three spheres and is tangent to them all.

Show all four spheres in the plan and in the right side elevation view.

Dash the invisible part of each sphere.

9-13-5. Scale: $\frac{1}{4}$ in. = 1 ft. 0 in.

A steel water tank, supported on a tower, has a hemispherical bottom 15 ft. in diameter. Show the approximate development for one of the steel plates for the bottom, using the meridian method. Divide the bottom into 12 sections.

9-13-6. Scale: $\frac{1}{8}$ in. = 1 ft. 0 in.

The metal dome of a building is to be made hemispherical in shape and 30 ft. in diameter. Show the approximate development of the steel plates, using the zone method. Divide the dome into six sections and assume that the plates are not over 24 ft. long.

9-13-7. Scale: 3 in. = 1 ft. 0 in.

A line *AB* is given and also a sphere 10 in. in diameter whose center is at *C*.

A is 5 in. to the right and 6 in. back of *C*, and 4 in. above *C*.
B is 9 in. to the right and 0 in. back of *C*, and 8 in. above *C*.
It is possible to have two planes tangent to the sphere and containing the line *AB*.

I. Show in the plan and front elevation these two tangent planes and their points of tangency with the sphere.

II. Find the size of the dihedral angle in degrees between these two tangent planes.

III. Find the true slope of each plane.

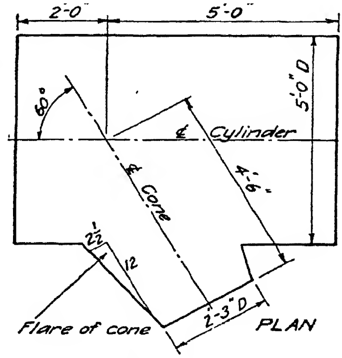


FIG. 9-14-1.

Group 14. Intersections

9-14-1. Scale: ½ in. = 1 ft. 0 in.
Conical reducer.
The cylinder and the cone are both surfaces of revolution. Their axes are both level but they do not intersect. The axis of the cone is 3 in. below the axis of the cylinder.

I. Show the complete curve of intersection in the plan.

II. Develop the conical surface.

III. Develop the cylindrical surface.

9-14-2. Scale: ½ in. = 1 ft. 0 in.
In problem 9-14-1 let the two axes be at the same level so they intersect.

I. Show the complete curve of intersection in the plan using the sphere method.

II. Develop the conical surface.

III. Develop the cylindrical surface.

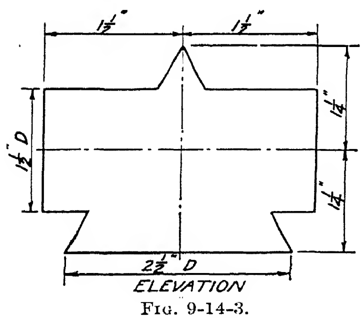


FIG. 9-14-3.

9-14-3. Scale: Actual size.
A cone of revolution and a cylinder of revolution intersecting. The axes are both in their true length in the view shown but they do not intersect. The axis of the cone is ⅜ in. behind the axis of the cylinder.

I. Show the complete curve of intersection in the given view and in the plan.

II. Develop the conical surface.

III. Develop the cylindrical surface.

9-14-4. Use the same scale and the same data as for problem 9-14-3, except that the axes are to be taken as intersecting.

I. Show the complete curve of intersection in the given view and also in the plan, using the sphere method.

II. Develop the conical surface.

III. Develop the cylindrical surface.

9-14-5. Scale: ¾ in. = 1 ft. 0 in.
A concrete wingwall as shown must have a hole left in it large enough to admit a level pipe having a 24 in. outside diameter.

The line X to Y is a straight line.

Draw complete plan, front elevation and right side elevation views.

9-14-6. Use the same scale and the same data as for problem 9-14-5, except that the line from X to Y is a part of a circle having its center directly in front of X and on the same level as X.

Draw complete plan, front elevation and right side elevation views.

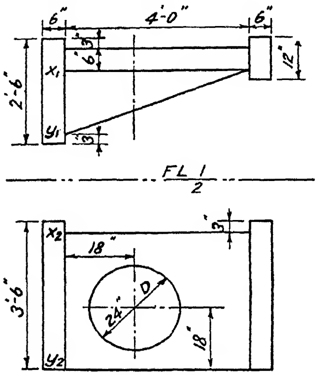


FIG. 9-14-5.

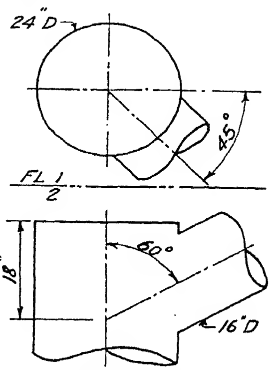


FIG. 9-14-7.

9-14-7. Scale: 1 in. = 1 ft. 0 in.

Two intersecting ventilating pipes as shown in Fig. 9-14-7.

I. Complete the curve of intersection of the two round pipes in the front elevation.

II. Develop a portion of the intersecting ends of the two round pipes.

9-14-8.[L] Scale: $1\frac{1}{2}$ in. = 1 ft. 0 in.

Double conical offset.

The three vertical ventilating pipes shown in part are to be connected by a double conical offset. Draw the given views, locating the center of the 20-in. pipe 4 in. from the left border and $\frac{1}{4}$ in. from the lower border and $1\frac{1}{2}$ in. from the upper border.

I. Show the complete line of intersection of the two cones. Show all hidden lines dashed and mark all tangent points accurately.

II. Develop the small cone. Show the outside of the metal up and make the seam as short as possible.

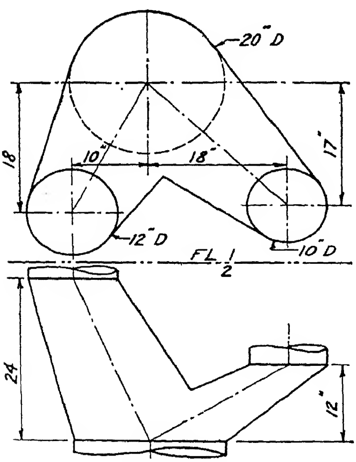


FIG. 9-14-8.

9-14-9.[L] Scale: Actual size.

Two intersecting cylinders having their bases in different planes.

One of the cylinders is a cylinder of revolution $1\frac{1}{2}$ in. in diameter.

I. Show the entire curve of intersection in both the given views.
II. Develop both of the cylinders from the base to the intersection line.

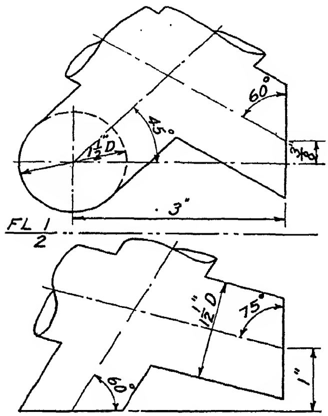


FIG. 9-14-9.

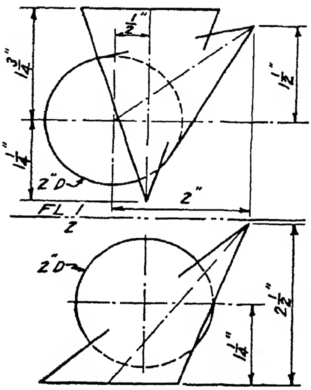
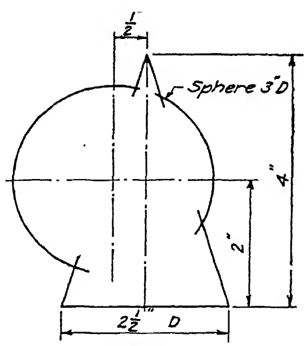


FIG. 9-14-10.

9-14-10. Scale: Actual size.

Two intersecting cones having their bases in different planes. One of the cones is a cone of revolution with a level axis.

I. Show the entire curve of intersection in both the given views.
II. Develop both the cones.



ELEVATION
FIG. 9-14-11.

9-14-11. Scale: Actual size.

A sphere intersected by a cone of revolution. The axis of the cone is vertical, it is ½ in. to the right and ½ in. behind the center of the sphere.

I. Show the entire curve of intersection in both the plan and front elevation. Use both circular and straight-line elements on the cone in determining points. (Two different methods.)

II. Develop the cone.

9-14-12. Scale: $\frac{3}{4}$ in. = 1 ft. 0 in.

A steel elbow is to be made as shown in the sketch. The main part of the elbow is a quarter of a torus and the small pipe is round.

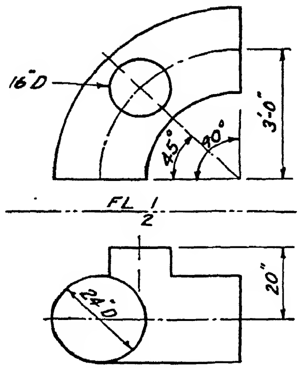


FIG. 9-14-12.

I. Show the entire curve of intersection in the front elevation.

II. Lay out a pattern that could be used for wrapping around and marking the round pipe so that it could be cut to fit exactly on to the elbow. It could then be welded in place.

Group 15. Offsets and Combination Surfaces

9-15-1. Scale: $\frac{1}{4}$ in. = 1 ft. 0 in.

Pipes *A* and *B* are both level and both 6 ft. in diameter. The axis of pipe *B* is 2 ft. $2\frac{1}{2}$ in. higher than the axis of pipe *A*. The two pipes have to be connected by an offset piece as shown.

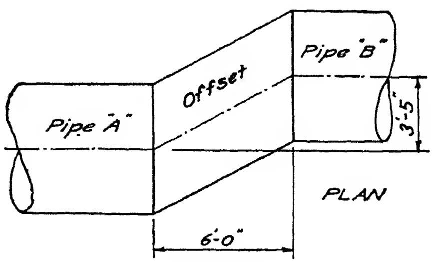


FIG. 9-15-1.

I. Find the true shape of the right section of the offset piece. Omit the front elevation and use the right side elevation instead.

II. Show the development of the offset.

9-15-2. Scale: $\frac{1}{4}$ in. = 1 ft. 0 in.

Use the same centerline layout as for problem 9-15-1. The pipe *A* is 6 ft. in diameter and the pipe *B* is 5 ft. in diameter.

Both axes are level. The connection will now be a conical offset.

Show the development of this conical offset.

9-15-3. Scale: 1 in. = 1 ft. 0 in.

Offset connection in a grain elevator between the rectangular bottom of a hopper and the top of a round pipe.

Show the complete development of the offset piece.

The surface is shown divided into four planes and four quarters of cones. Each quarter of the circle is a quarter of the base of a cone having a vertex at one of the four corners of the hopper.

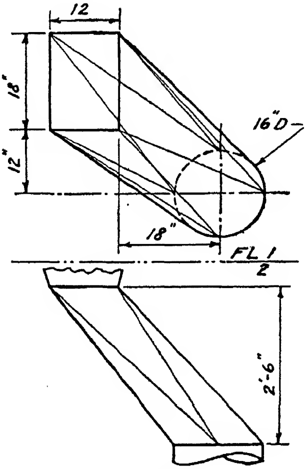


FIG. 9-15-3.

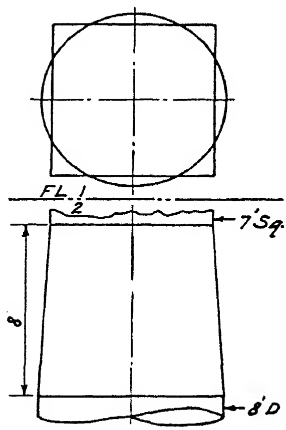


FIG. 9-15-4.

9-15-4. Scale: 3/8 in. = 1 ft. 0 in.

Smoke-pipe transition from a round to a square stack.

Show the development of one quarter of the transition. The rest is similar.

Follow the suggestion offered in problem 9-15-3 regarding dividing up the surface into cones and planes.

Group 16. Miscellaneous

9-16-1. Scale: 1/8 in. = 1 ft. 0 in.

The points *A*, *B*, and *C* are three located points on the face of a dam. *A* is 4 ft. west and 17 ft. north of *C*, and at an elevation of 1,826 ft. *B* is 16 ft. east and 4 ft. north of *C*, and at an elevation of 1,823 ft. *C* is at an elevation of 1,810 ft.

A is the center of an opening 12 ft. in diameter and running perpendicular to the face of the dam for a depth of 4 ft.

B is the center of an opening 12 ft. square and running perpendicular to the face of the dam for a depth of 8 ft. The square opening is also boarded

out from the face of the dam for a distance of 4 ft. Two sides of the square opening are level.

Show the two openings in the plan and front elevation views.

9-16-2. Scale: 1 in. = 10 ft.

The drawing shows the map of a corner lot. It is desired to level off this lot at an elevation of 110 ft. so that each of the two terraces will have a 1:1 slope from the level top to the two property lines.

Show where the limiting edge of the level part of the lot would be and where the two terraces would intersect.

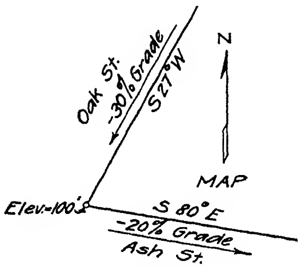


FIG. 9-16-2.

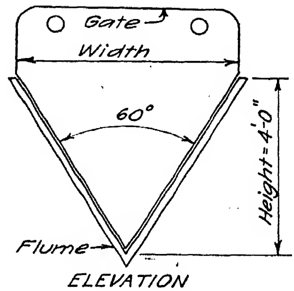


FIG. 9-16-3.

9-16-3. Scale: ¾ in. = 1 ft. 0 in.

A triangular-shaped flume and a steel gate. One gate for controlling the water flow in this flume was shipped the correct height but the bottom angle was made 50 instead of 60 degrees.

A second gate was shipped the correct height but the bottom angle was made 70 instead of 60 degrees.

Show how both gates could be used temporarily without alteration until the correct gate arrived.

9-16-4.[L] Scale: ¼ in. = 1 ft. 0 in.

The centerlines of two 8-in. pipes are *AB* and *BC*, and the pipes meet at *B*.

B is 13 ft. due east of *A*, and 6 ft. below *A*.

C is 6 ft. north and 6 ft. east of *A*, and 2 ft. above *A*.

From *B* a 12-in. pipe continues down, so that its centerline has a 15-degree slope and makes equal angles with *AB* and *BC*.

Find:

I. The true bearing of the 12-in. pipe.

II. The angle between the two 8-in. pipes.

III. The angle between the 12-in. pipe and the plane of the 8-in. pipes.

Show a free-hand sketch of the special flanged fitting required at *B*, indicating on the sketch the values in II and III.

9-16-5.[L] Scale: 1 in. = 50 ft.

A penstock carrying water from the forebay to the power house. From *A* the pipe is level through the forebay wall.

Find:

I. True length of the centerline from *A* to *B* and from *B* to *C*.

II. True angle (in degrees) of the pipe bends at A and B.

III. True angle between the part BC and the wall of the powerhouse.

IV. True shape of the hole in the power-house wall.

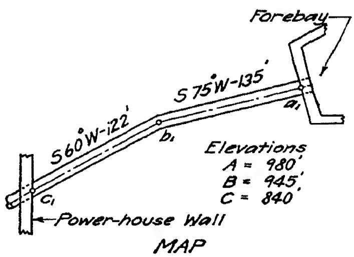


Fig. 9-16-5.

9-16-6. Scale: 1 in. = 40 ft.

A pipe line crossing a canal.

Using the centerlines only, find:

I. The true length of each of the three portions shown.

II. The true size of the four bend angles at A, B, C, and D.

9-16-7. Scale: ½ in. = 1 ft. 0 in.

Trap and sewer problem, as taken from a nationally known trade magazine.

The stock trap and stock branch are as shown.

The vertical centerline of the trap is a fixed distance away from the centerline of the sewer as shown in the plan.

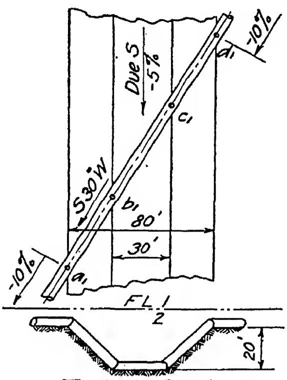


Fig. 9-16-6.

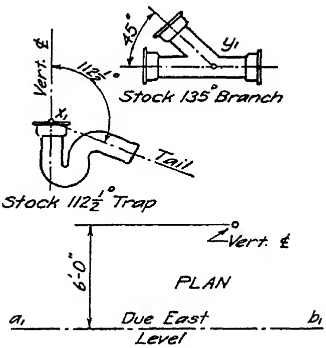


Fig. 9-16-7.

AB is the main sewer, which bears due east and it has so little slope that it may be considered level. It is desired to determine where to place the branch along the main sewer and the trap along the vertical centerline so that they may be connected by a straight piece of pipe.

Find:

I. The centerline length of the branch pipe from X to Y.

II. The distance Y would be east or west of X.

III. The bearing of the branch line.

IV. The elevation of X above the main sewer.

9-16-8. Scale: ½ in. = 1 ft. 0 in.

Use the same trap and branch and the same general setting as in problem 9-16-7, changing only the relative position of the main sewer and the trap.

For this problem, as shown in Fig. 9-16-8, the main sewer bears S 75° W, falls 15 degrees and is 4 ft. from the vertical centerline of the trap.

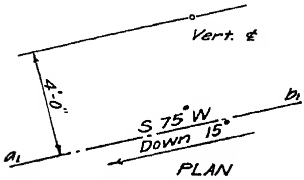


FIG. 9-16-8.

It is desired to determine where to place the branch and the trap so that they may be connected with a straight pipe.

Find:

I. The centerline length of the branch pipe from X to Y.

II. The distance Y would be south and west of X on the map.

III. The bearing of the branch pipe.

IV. The elevation of X above Y to give the height at which the trap will have to be placed.

9-16-9.[T] Scale: ¾ in. = 1 ft. 0 in.

Three 8- by 12-in. floor beams are located as shown, with the 12-in. faces vertical.

The beams are all level but the floor is inclined. The given hopper is shown with a dash line.

Show where to cut the beams so as to allow the hopper to set down through them until the top of the hopper is 12 in. above the top of the highest beam.

Turn the hopper at the angle shown.

Show the hopper with a fine dash line just to indicate where it would go.

Show the beam cuts in both views and cross-hatch the visible portion of the cuts.

Solve by one method and check by another method.

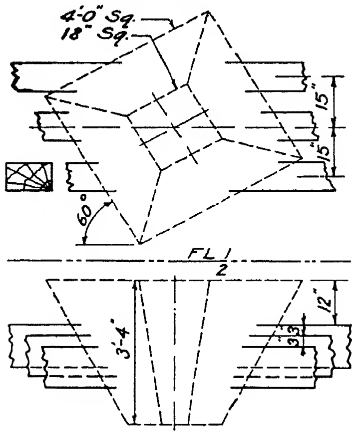


FIG. 9-16-9.

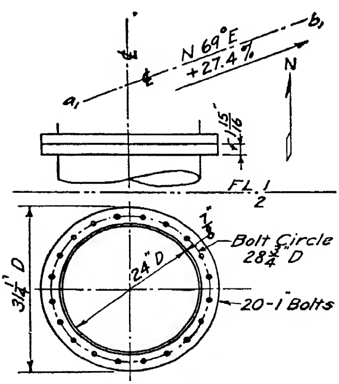


FIG. 9-16-10.

9-16-10. Scale: 1 in. = 1 ft. 0 in.

An elbow is required to connect the given level pipe running due north with the pipe AB which bears N 69° E and rises on a 27.4-per cent grade

I. Find the true angle of the bend for the elbow.

II. Locate the bolt holes in the connecting flange of the elbow so they will coincide with the bolt holes as shown on the level pipe flange. Measure the angle the centerline must be shifted.

III. Make a detail shop drawing of the elbow.

9-16-11. Two 8-in. pipes going into a 12-in. pipe are located as shown. Design a special flanged *Y* lateral for this condition.

The bolt holes are to be drilled so as to allow standard valves to be attached to the 8-in. or the 12-in. flanges and have each valve stem in a vertical plane containing the centerline of its respective pipe.

The bolt holes on the standard straddle the centerlines.

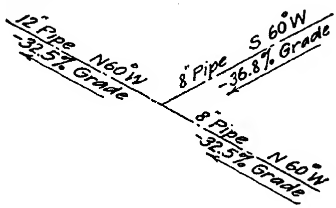


FIG. 9-16-11.

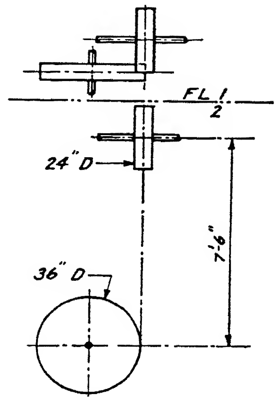


FIG. 9-16-12.

I. Solve all the necessary angles.

II. On a separate sheet make a complete shop drawing of the lateral. Scale: 1½ in. = 1 ft. 0 in.

The following information is taken from a table of standard fittings and it is to be used in dimensioning the lateral.

	8-in. pipe	12-in. pipe
Thickness of pipe	0.46	0.54
Thickness of flange	1⅛	1¼
Diameter of flange	13½	19
Diameter of bolt circle	11¾	17
Size of bolt holes	⅞	1
Number of bolt holes	8	12
Distance from center to outside face of flange	15¼	6

9-16-12.[L] Scale: ½ in. = 1 ft. 0 in.

Belt and pulley problem.

In order for a belt to run on to a pulley it must be moving in the midplane of the pulley before it reaches the pulley. With the two pulleys located as shown in the sketch, it is impossible for a belt to run in either direction.

You are to install a guide or idler pulley (24 in. in diameter) to guide the belt so that it will operate in either direction over the given pulleys.

Show this new pulley and the belt in both the given views.

Find the arc of contact between the belt and the idler pulley.

All pulleys have a 6-in. face and the crown may be neglected. All shafts are 2 in. in diameter and the belt is 4 in. wide.

Suggestion. Choose any point on the line of intersection of the midplanes of the two pulleys.

From this point draw a line tangent to each pulley. These two lines determine the midplane of the required idler pulley.

9-16-13.[L] Scale: ⅛ in. = 1 ft. 0 in. Trestle problem.

A railroad track, running due east on a trestle, crosses over a highway 12 ft. wide bearing N 30° W. Both are level. The centerline of the highway

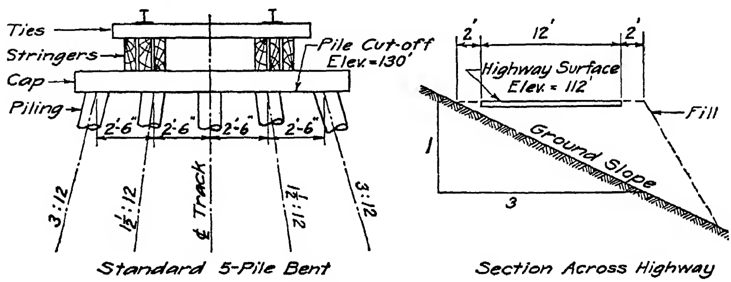


Fig. 9-16-13.

is parallel to a level line on the ground. The ground has a slope of 1:3 and is low in the northeast quarter.

Use two standard 5-pile bents as shown to support the stringers carrying the track over the highway. The plane of each bent is placed perpendicular to the centerline of the track. At the level of the highway surface the centerline of the nearest pile must not be closer than 4 ft. to the edge of the highway.

Find the shortest possible span between the centerlines of the two bents and the true length of all piling from the cut-off to the ground. Disregard the fill.

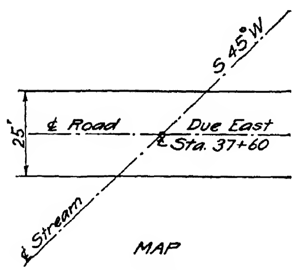


Fig. 9-16-14.

9-16-14. Scale: 1 in. = 30 ft. Culvert problem.

The bearing of a road is due east with a rising grade of 10 per cent toward the east. Station 37 + 60 on the road is just 25 ft. above the centerline of a stream flowing S 45° W on a 15-per cent grade.

A culvert 8 ft. square, outside measurement, is to be so placed that it will carry the stream under the road fill. The centerline of the bottom plane of the culvert coincides with the centerline of the stream.

The road surface is 25 ft. wide. The side slopes of the road are 1 vertical to 1½ horizontal, but measured in a vertical plane at right angles to the plan view of the centerline of the road. (This gives a slope that is not theoretically correct, but it is the actual way it would be surveyed in road work.)

Find the true length of the centerline of the top plane of the culvert between the two side slopes.

9-16-15.[L] Scale: ¾ in. = 1 ft. 0 in.

The line AB is the centerline of a hawse pipe for a boat. The pipe is a cylindrical cast-steel pipe 12 in. inside diameter with walls 1 in. thick. A flange 1 in. thick and 4 in. wide is cast on the lower or outboard end.

I. Make a completely dimensioned shop drawing of the pipe and flange cast in one piece ready to go into place.

II. Make a shop drawing of the deck flange to fit flush against the deck end of the pipe.

III. Make a dimensioned drawing of a template to be used for cutting this pipe out of a stock pipe. The template would be wrapped around the pipe. In this case the flange would have to be welded on.

For the sake of simplifying this problem, assume the small portion of the deck and the sides shown are plane surfaces.

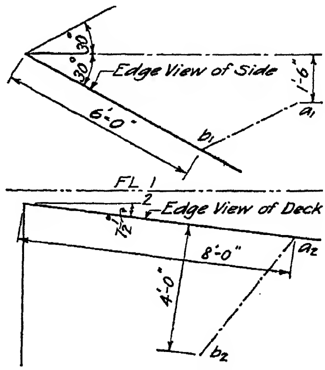


Fig. 9-16-15.

9-16-16.[L] Scale: 1 in. = 1 ft. 0 in.

Connect the two square openings in the walls by the largest possible grain spout made of steel plate. Lay out a development of this spout and dimension it completely. Allow a 2-in. lap all around for riveting (except where two edges come together).

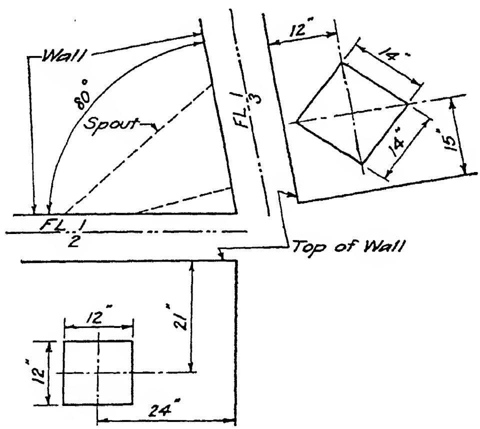


Fig. 9-16-16.

Use the revolution method for finding the true length of all plane intersection lines.

9-16-17. Domebrick problem as taken from actual practice.

The firebrick lining of the dome of the water-gas generator shown consists of 24 courses. The first 16 courses form the spherical part, the next

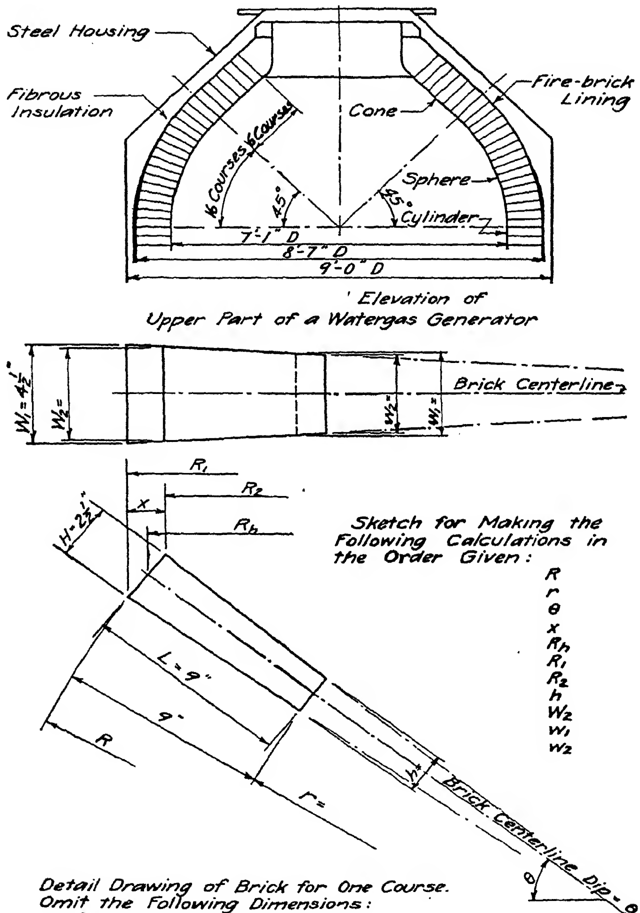


Fig. 9-16-17.

6 courses the cone part, and the last 2 courses the top opening. The firebrick used in the spherical part are called domebrick.

A domebrick is a combination arch-wedge-key brick of the basic size $2\frac{1}{2}$ in. high, $4\frac{1}{2}$ in. wide and 9 in. long. It may be thought of as being cut from a standard brick of this size.

The undersides of the brick in the first course of the dome lie in a horizontal plane passing through the sphere center; while the upper brick surfaces form a right circular cone with its vertex at the sphere center, and having a very small vertical altitude.

The undersides of the brick in the next course coincide with this cone; the upper brick surfaces form a cone with greater vertical altitude, until finally the gradually increasing brick dip reaches its predetermined maximum of 45 degrees after the 16th course is laid up. Each course is laid up with one size of domebrick, a different size being required for each course.

All domebrick surfaces are planes. Bricks are laid up tightly without allowance for joints. As seen in Fig. 9-16-17, one dimension of any brick will be 9 in., another 4½ in., and another 2½ in. Any other dimension may be scaled from a full-size layout, or calculated mathematically by similar triangle methods. Bear in mind that all extension lines and great or small circle radii pass through the sphere center in the elevation or in the plan view. The order for making these calculations is indicated in the drawing.

Make a detailed dimensioned drawing of a 14th course domebrick. Scale: 3 in. = 1 ft. 0 in. Place the views as shown and calculate all necessary dimensions to the nearest $\frac{1}{32}$ in.

9-16-18.[L] Drift-barrier problem.

The drift barrier used in this problem was actually constructed in a western river to keep drift wood away from the intakes to the water lines. Figure 9-16-18 shows a partial plan view of the 14 piers in which heavy rods were anchored for the purpose of holding the wire barriers. A half elevation of one of the barriers is also shown.

I. Draw a complete plan and front elevation of a typical pier. Scale: $\frac{3}{16}$ in. = 1 ft. 0 in. Place these views on the left side of the sheet.

II. In these views show in position the 10 eyebars, each bar lying in a horizontal plane with the vertex of its bend on the line AB (shown in the front elevation). The lowest horizontal plane is 1 ft. above low-water elevation and the planes are 1 ft. apart. Locate the exact points at which each end of each bar comes through the sloping concrete wall of the pier.

III. Using a scale of ½ in. = 1 ft. 0 in., detail one typical eyebar and make a table of lengths for all the bars. Allow each end of each bar to project 6 in. out from the concrete.

9-16-19. Scale: 3 in. = 1 ft. 0 in.

Draw a left-hand screw conveyor with a convolute blade on a 5-in. shaft and inside a 12-in. cylinder. The lead is 8 in.

Lay out the development for one turn of the blade. Calculate the number of turns of the blade that could be made in one piece of metal.

9-16-20.[L] Scale: ½ in. = 1 ft. 0 in.

Dead-end tower for an electric power line.

This is an actual problem that was given to a draftsman who was working for a power company.

The drawing shows part of a three-pole tower for a 167,000-volt line. The dash line from A and C to the frame represents a string of insulators between the conductor and the frame.

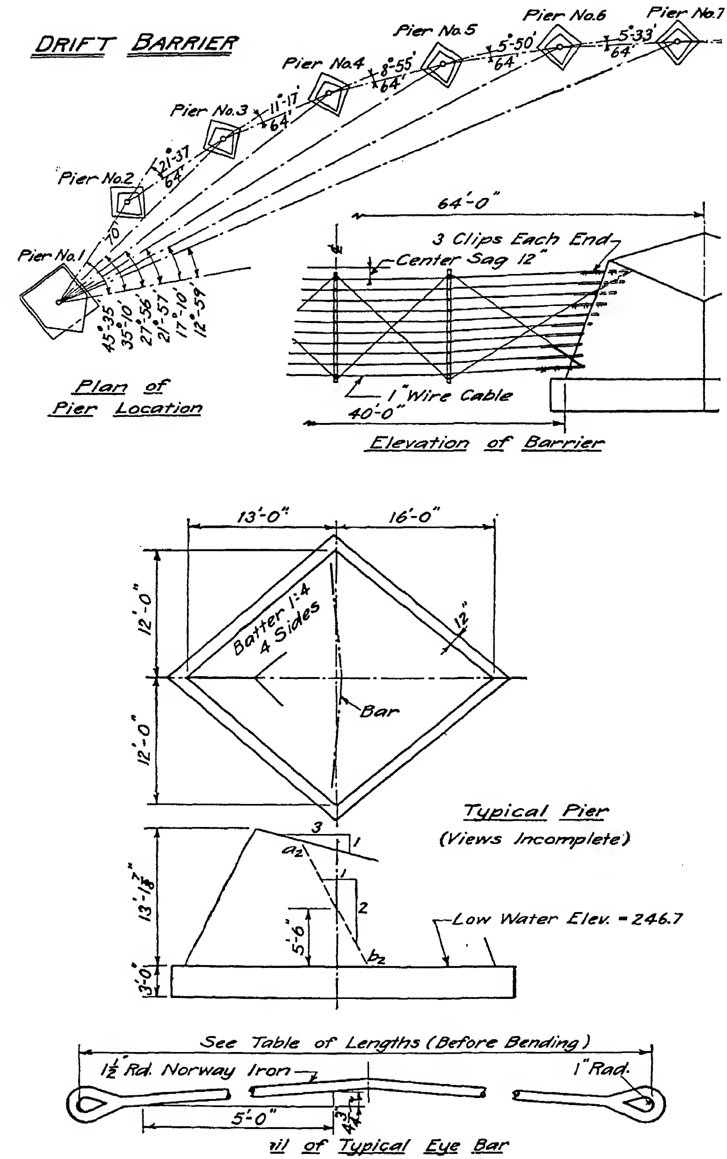

DRIFT BARRIER
Pier No.1
Pier No.2
Pier No.3
Pier No.4
Pier No.5
Pier No.6
Pier No.7
21°-37' 64'
11°-17' 64'
8°-55' 64'
5°-50' 64'
5°-33' 64'
70'
45°-35'
35°-10'
27°-56'
21°-57'
17°-10'
12°-59'
Plan of Pier Location
64'-0"
3 Clips Each End
Center Sag 12"
1" Wire Cable
40'-0"
Elevation of Barrier
13'-0"
16'-0"
12'-0"
12'-0"
Batter 1:4 4 Sides
12"
Bar
Typical Pier
(Views Incomplete)
13'-1⅞"
5'-6"
3'-0"
3
1
a_2
1
2
b_2
Low Water Elev. = 246.7
See Table of Lengths (Before Bending)
1½" Rd. Norway Iron
1" Rad.
5'-0"
3"
4½"
il of Typical Eye Bar

FIG. 9-16-18.

Between A and C the conductor is allowed to sag in a loop, as shown in the end elevation; this is called a jumper.

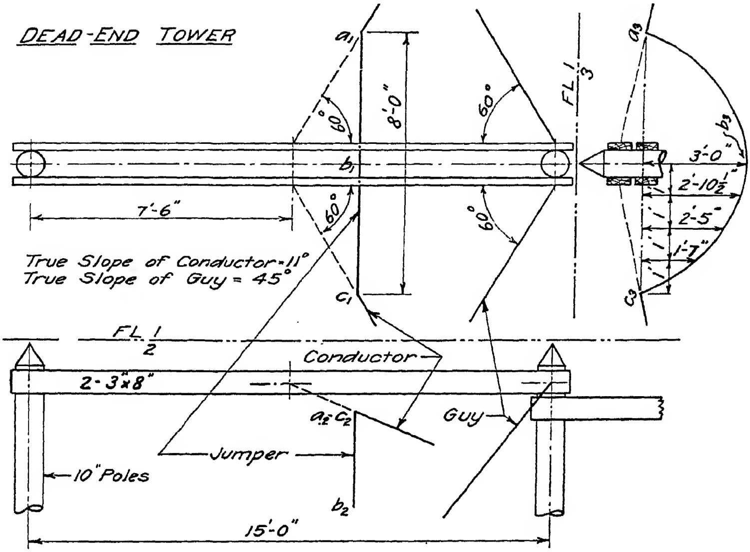


FIG. 9-16-20.

It is necessary to have a definite clearance between the nearest points on the live wire and the guy wire. This presents the three conditions which require checking.

I. Find the closest distance between the jumper and the guy wire.

II. Find the closest distance between the straight conductor and the guy wire.

III. Find the closest distance between the guy wire and the jumper when the wind blows it 60 degrees away from its vertical position.

Scale all distances accurately. Assume that the guy wire and the conductor (except the jumper) are both straight lines for the short length which is used.

9-16-21. Scale: 3 in. = 1 ft. 0 in.

A small hand-operated concrete mixer for laboratory use has a horizontal cylinder 12 in. in diameter. The blades are attached to arms on the shaft at the angle shown. The blades are plane surfaces and they have a true slope of 12 (vertical) to 5 (horizontal).

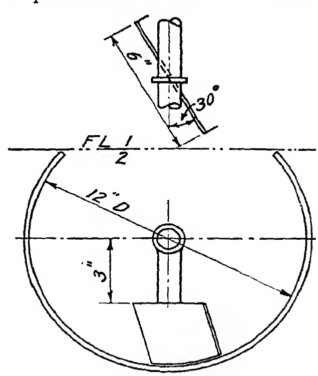


FIG. 9-16-21.

Make a layout of one metal blade designed so as to give 1/8 in. clearance from the cylinder for the entire length of the blade.

9-16-22. Scale: 3/16 in. = 1 ft. 0 in.

The setting for this problem is taken direct from a problem that arose in the yards of a transcontinental railway.

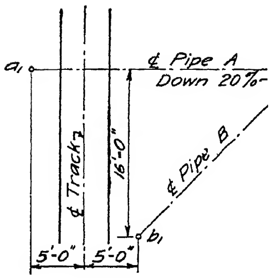


Fig. 9-16-22.

The centerline of a track bears due north and is level. The points A and B, located as shown, are the centers of two catch basins.

They are both 12 in. below the centerline of the track.

From A a drainage pipe bears due east under the track and has a falling grade of 20 per cent. A second drain pipe from B is to connect into the pipe from A.

The engineer wishes you to furnish him the following information:

I. In order to use a stock 45-degree connection, find the length of the A pipe, the length of the B pipe, and the bearing of the B pipe.

II. In order for the two drain pipes to have the same grade, find the length of the A pipe, the length of the B pipe, the bearing of the B pipe, and the angle for the special connection required.

9-16-23.[L] Scale: 3 in. = 1 ft. 0 in.

The following problem is almost an exact duplicate of an actual problem that had to be solved recently in fitting a tube against the aileron of an airplane. The data has been changed slightly just for convenience in making the solution.

A level metal tube must be cut so it fits exactly flush against the sloping plane $ABCD$, shown in Fig. 9-16-23. Find the angle at which the pipe must be cut off. Make a development of the pipe which may be used as a template to wrap around the pipe for cutting the end next to the plane.

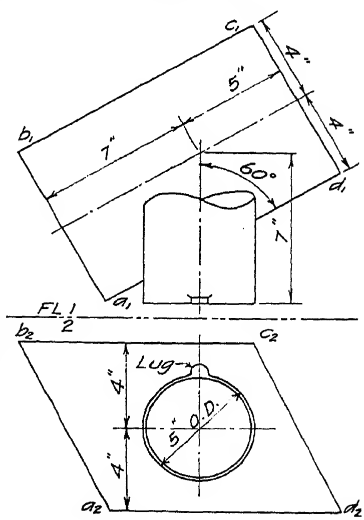


Fig. 9-16-23.

A lug, shown on the outer end of the pipe, is in the same vertical plane as the centerline of the pipe. Locate the centerline of this lug on the development so the lug will remain in the correct position after the pipe is in place.

9-16-24.[L] Scale: 1 in. = 20 ft. Two pipe lines, AB and CD.

B is 57 ft. south and 42 ft. east of A and 22 ft. above A.

C is 59 ft. south and 5 ft. west of A and 2 ft. below A.

D is 21 ft. south and 40 ft. east of A and 15 ft. below A.

Locate the shortest possible pipe having a true slope of 45 degrees and connecting the pipes AB and CD. Find the bearing and true length of this pipe.

Group 17. Mining Problems

9-17-1. Scale: 1 in. = 50 ft.

A, B, and C are known points of outcrop on a vein of ore.

B is 125 ft. due south of A and 100 ft. below A.

C is 100 ft. south and 100 ft. east of A and 25 ft. below A.

Find:

I. Strike of the vein.

II. True dip of the vein.

III. True slope in going from A to B direct.

9-17-2. A vein of ore is cut off by a vertical cliff which runs due east and west. The streak of the exposed ore as it is seen on this cliff appears to dip down 52 degrees in an easterly direction. The strike of the vein is known to be N 45° E. Find the true dip of the vein.

NOTE: An easterly dip does not mean that a vein dips exactly due east. It does mean that the vein is low in the eastern portion of the map and high in the western portion. Or, any point on the vein is lower than any other point on the vein from which it is due east.

9-17-3. An inclined mine shaft AB, and a vertical shaft. Scale: 1 in. = 30 ft.

A is 60 ft. north and 56 ft. west of B and 40 ft. above B. The vertical shaft is 35 ft. due north of B. The bottom, C, of the vertical shaft is 45 ft. higher than B.

Find:

I. The bearing, true length, and per cent grade of the shortest shaft to connect C with AB.

II. The bearing, true length, and per cent grade of the steepest shaft to connect C with AB.

9-17-4. Scale: 1 in. = 100 ft.

A is a point of outcrop of ore. A borehole is started at M, which is 150 ft. west and 125 ft. south of A and 25 ft. below A. The hole bears S 60° E and falls 45 degrees. Ore is struck after boring 130 ft. A second hole is started at N, which is 150 ft. east and 75 ft. south of A and 25 ft. below A. This hole bears S 45° W and falls 60 degrees. Ore is struck after going 190 ft. Neglect the thickness of the vein.

Find:

I. Strike of ore vein.

II. Dip of ore vein.

9-17-5. Scale: 1 in. = 10 ft.

A vein of ore whose strike is S 60° E is known to be 9 ft. thick. A level tunnel bearing due south from A pierces the upper surface of this vein after going 15 ft. and the lower surface after going an additional 25 ft. If the tunnel from A had been driven with the same bearing and a −30 per cent grade, what would have been the distance to the vein?

9-17-6. Scale: 1 in. = 10 ft.

Three short holes are drilled through a vein of ore from a point of outcrop *A* on the upper surface of the vein. One hole bears due north on a rising 20 per cent grade and shows 20 ft. of thickness to the vein; a level hole bears due west and shows 10 ft. of thickness; the third hole is vertical and shows 15 ft. of thickness.

Find the strike, dip, and real thickness of the vein.

9-17-7. Scale: 1 in. = 40 ft.

A vein of ore is located by three points of outcrop, *A*, *B*, and *C*. *B* is 100 ft. due north of *A* and 80 ft. higher than *A*. *C* is 90 ft. due east of *B* and 60 ft. lower than *B*. A vertical hole drilled at *A* shows the vein to be 25 ft. thick.

Find the strike, dip, and real thickness of the vein.

9-17-8. Scale: 1 in. = 20 ft.

A and *B* are two points on the upper surface of a vein of ore and *X* and *Y* are two points on the lower surface.

B is 50 ft. east and 35 ft. south of *A* and 25 ft. higher than *A*.

X is 60 ft. due east of *A* and 30 ft. higher than *A*.

Y is 30 ft. due east of *A* and 5 ft. lower than *A*.

Find the strike, dip, and thickness of the vein.

9-17-9. Scale: 1 in. = 40 ft.

A vein of ore is known to have an easterly* dip and to be 20 ft. thick. A level borehole bears due west from *A* and intersects the upper and lower faces of the vein at 35 ft. and 70 ft., respectively, from *A*. *X* is a point of outcrop on the upper face of the vein and is 50 ft. due north of *A* and 25 ft. higher than *A*.

Find the strike and dip of the vein.

9-17-10. Scale: 1 in. = 20 ft.

A vertical borehole from *A* intersects the upper surface of a vein at a depth of 8 ft. and the lower surface at a depth of 30 ft. from *A*. A second borehole is sunk from a point *B* which is 28 ft. due north of *A* and 13 ft. lower in elevation than *A*. This hole bears due east, falls 60 degrees and intersects the upper surface of the vein at 30 ft. and the lower surface at 50 ft., distances being measured from *B* along the hole.

Find the strike, dip, and thickness of the vein.

9-17-11. Scale: 1 in. = 20 ft.

A vein of ore is known to have a westerly* dip and a thickness of 14 ft. Two vertical boreholes are sunk at the points *A* and *M*. *A* is 40 ft. west and 23 ft. north of *M* and on the same level as *M*. The hole at *M* reached the vein at a depth of 10 ft. and the one at *A* at a depth of 30 ft. Both holes showed a thickness of 20 ft. through the vein.

Find the strike and dip of the vein.

* See note under problem 9-17-2.

APPENDIX

A1. Trammel Method for Drawing an Ellipse.

An ellipse may be quickly and accurately drawn by the use of a trammel when the lengths of the major and minor axes are known. The best trammel to use is a thin card or paper with a very straight edge. Along this straight edge mark some point *A*, as in Fig. A 1, which is the point which will draw the ellipse.

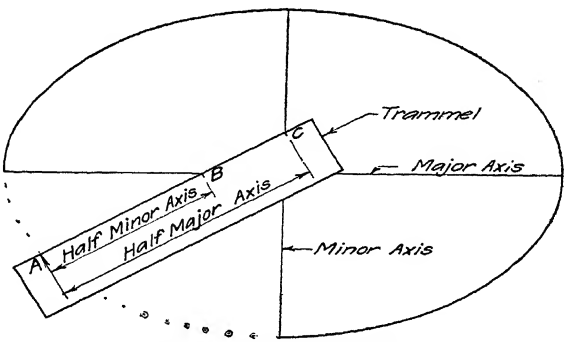


Fig. A 1.—Trammel method.

Mark the point *B* at a distance of half the minor axis from *A*. Also mark the point *C* at a distance of half the major axis from *A*. Lay the trammel across the two axes as shown so the points *B* and *C* touch the axes and make a pencil mark on the drawing opposite the point *A*. This will be one point on the ellipse. In the same manner lay the trammel in several other positions until sufficient points are marked to establish the entire curve. Then draw the ellipse with a French curve.

A2. Sheet-metal Workers Method for Finding True Lengths of Elements.

Sheet-metal workers use a special short-cut method for finding the true lengths of several elements in preparation for developing a curved surface. This method is well worth knowing because it

is convenient and accurate and because it saves much confusion on the drawing.

Case 1. When all the elements have the same difference in elevation between their two ends. See Fig. A 2.

Lay out the lines *AB* and *CD* at right angles to each other. Make *AB* equal in length to the difference in elevation between

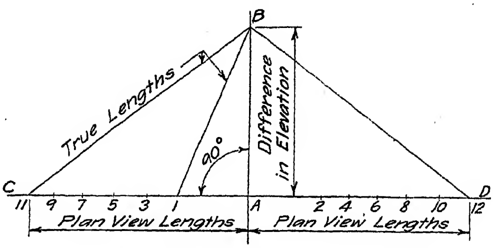


Fig. A 2.—Special method for true lengths.

the two ends of any element. From *A* measure along the line *CD* the plan view length of each element and number each point thus obtained with the element number. These points may be measured to the left or to the right of *AB*. For convenience in Fig. A 2, the even-numbered elements are shown on the right and the odd-numbered on the left. From *B* to these ends just

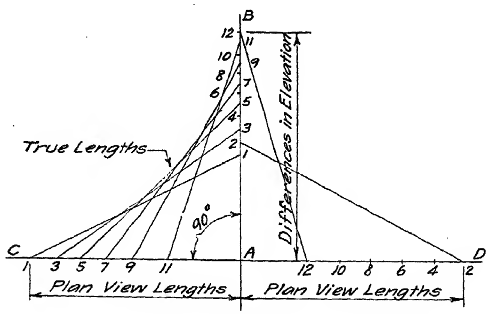


Fig. A 3 -Special method for true lengths.

numbered along *CD* are the true lengths of the elements, which do not actually need to be drawn. The true lengths of all the elements needed are on this diagram and may be transferred to the development with dividers.

Case 2. When all the elements do not have the same difference in elevation between their two ends. See Fig. A 3.

The general procedure is the same as for Case 1 except that the differences in elevation, since they are all different, are laid out from A towards B and must also be numbered with the number of the element. The distances from the numbered points along AB to the corresponding numbered points along CD are the true lengths of the elements.

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